Chapter 2 Selected Problem Solutions

Section 2-2

2-43. 3 digits between 0 and 9, so the probability of any three numbers is 1/(10*10*10);
3 letters A to Z, so the probability of any three numbers is 1/(26*26*26); The probability your license plate is chosen is then $(1/10^3)*(1/26^3) = 5.7 \times 10^{-8}$

Section 2-3

2-49. a) $P(A') = 1- P(A) = 0.7$
b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3+0.2 - 0.1 = 0.4$
c) $P(A' \cap B) + P(A \cap B) = P(B)$. Therefore, $P(A' \cap B) = 0.2 - 0.1 = 0.1$
d) $P(A) = P(A \cap B) + P(A \cap B')$. Therefore, $P(A \cap B') = 0.3 - 0.1 = 0.2$
e) $P((A \cup B)') = 1 - P(A \cup B) = 1 - 0.4 = 0.6$
f) $P(A' \cup B) = P(A') + P(B) - P(A' \cap B) = 0.7 + 0.2 - 0.1 = 0.8$

Section 2-4

2-61. Need data from example
a) $P(A) = 0.05 + 0.10 = 0.15$
b) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.04 + 0.07}{0.72} = 0.153$
c) $P(B) = 0.72$
d) $P(B|A) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.04 + 0.07}{0.15} = 0.733$
e) $P(A \cap B) = 0.04 +0.07 = 0.11$
f) $P(A \cup B) = 0.15 + 0.72 – 0.11 = 0.76$

2-67. a) P(gas leak) = (55 + 32)/107 = 0.813
b) P(electric failure|gas leak) = (55/107)/(87/102) = 0.632
c) P(gas leak| electric failure) = (55/107)/(72/107) = 0.764

Section 2-5

2-73. Let F denote the event that a roll contains a flaw.
Let C denote the event that a roll is cotton.

$$\begin{aligned} P(F) &= P(F|C)P(C) + P(F|C')P(C') \\ &= (0.02)(0.70) + (0.03)(0.30) = 0.023 \end{aligned}$$

2-79. Let A denote a event that the first part selected has excessive shrinkage.
Let B denote the event that the second part selected has excessive shrinkage.
a) $P(B)= P(B|A)P(A) + P(B|A')P(A')$
$= (4/24)(5/25) + (5/24)(20/25) = 0.20$
b) Let C denote the event that the second part selected has excessive shrinkage.

$$\begin{aligned} P(C) &= P(C|A \cap B)P(A \cap B) + P(C|A \cap B')P(A \cap B') \\ &\quad + P(C|A' \cap B)P(A' \cap B) + P(C|A' \cap B')P(A' \cap B') \\ &= \frac{3}{23}\left(\frac{2}{24}\right)\left(\frac{5}{25}\right) + \frac{4}{23}\left(\frac{20}{24}\right)\left(\frac{5}{25}\right) + \frac{4}{23}\left(\frac{5}{24}\right)\left(\frac{20}{25}\right) + \frac{5}{23}\left(\frac{19}{24}\right)\left(\frac{20}{25}\right) \\ &= 0.20 \end{aligned}$$

Section 2-6

2-87. It is useful to work one of these exercises with care to illustrate the laws of probability. Let H_i denote the event that the ith sample contains high levels of contamination.

a) $P(H_1' \cap H_2' \cap H_3' \cap H_4' \cap H_5') = P(H_1')P(H_2')P(H_3')P(H_4')P(H_5')$

by independence. Also, $P(H_i') = 0.9$. Therefore, the answer is $0.9^5 = 0.59$

b) $A_1 = (H_1 \cap H_2' \cap H_3' \cap H_4' \cap H_5')$

$A_2 = (H_1' \cap H_2 \cap H_3' \cap H_4' \cap H_5')$

$A_3 = (H_1' \cap H_2' \cap H_3 \cap H_4' \cap H_5')$

$A_4 = (H_1' \cap H_2' \cap H_3' \cap H_4 \cap H_5')$

$A_5 = (H_1' \cap H_2' \cap H_3' \cap H_4' \cap H_5)$

The requested probability is the probability of the union $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$ and these events are mutually exclusive. Also, by independence $P(A_i) = 0.9^4(0.1) = 0.0656$. Therefore, the answer is 5(0.0656) = 0.328.

c) Let B denote the event that no sample contains high levels of contamination. The requested probability is P(B') = 1 - P(B). From part (a), P(B') = 1 - 0.59 = 0.41.

2-89. Let A denote the event that a sample is produced in cavity one of the mold.

a) By independence, $P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5) = (\frac{1}{8})^5 = 0.00003$

b) Let B_i be the event that all five samples are produced in cavity i. Because the B's are mutually exclusive, $P(B_1 \cup B_2 \cup ... \cup B_8) = P(B_1) + P(B_2) + ... + P(B_8)$

From part a., $P(B_i) = (\frac{1}{8})^5$. Therefore, the answer is $8(\frac{1}{8})^5 = 0.00024$

c) By independence, $P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5') = (\frac{1}{8})^4(\frac{7}{8})$. The number of sequences in which four out of five samples are from cavity one is 5. Therefore, the answer is $5(\frac{1}{8})^4(\frac{7}{8}) = 0.00107$.

Section 2-7

2-97. Let G denote a product that received a good review. Let H, M, and P denote products that were high, moderate, and poor performers, respectively.

a)

$$\begin{aligned} P(G) &= P(G|H)P(H) + P(G|M)P(M) + P(G|P)P(P) \\ &= 0.95(0.40) + 0.60(0.35) + 0.10(0.25) \\ &= 0.615 \end{aligned}$$

b) Using the result from part a.,

$$P(H|G) = \frac{P(G|H)P(H)}{P(G)} = \frac{0.95(0.40)}{0.615} = 0.618$$

c) $$P(H|G') = \frac{P(G'|H)P(H)}{P(G')} = \frac{0.05(0.40)}{1 - 0.615} = 0.052$$

Supplemental

2-105. a) No, $P(E_1 \cap E_2 \cap E_3) \neq 0$

b) No, $E_1' \cap E_2'$ is not $\varnothing$

c) $$\begin{aligned} P(E_1' \cup E_2' \cup E_3') &= P(E_1') + P(E_2') + P(E_3') - P(E_1' \cap E_2') - P(E_1' \cap E_3') - P(E_2' \cap E_3') \\ &\quad + P(E_1' \cap E_2' \cap E_3') \\ &= 40/240 \end{aligned}$$

d) $P(E_1 \cap E_2 \cap E_3) = 200/240$
e) $P(E_1 \cup E_3) = P(E_1) + P(E_3) - P(E_1 \cap E_3) = 234/240$
f) $P(E_1 \cup E_2 \cup E_3) = 1 - P(E_1' \cap E_2' \cap E_3') = 1 - 0 = 1$

2-107. Let A_i denote the event that the ith bolt selected is not torqued to the proper limit.
a) Then,

$$P(A_1 \cap A_2 \cap A_3 \cap A_4) = P(A_4 | A_1 \cap A_2 \cap A_3) P(A_1 \cap A_2 \cap A_3)$$
$$= P(A_4 | A_1 \cap A_2 \cap A_3) P(A_3 | A_1 \cap A_2) P(A_2 | A_1) P(A_1)$$
$$= \left(\frac{2}{17}\right)\left(\frac{3}{18}\right)\left(\frac{4}{19}\right)\left(\frac{5}{20}\right) = 0.282$$

b) Let B denote the event that at least one of the selected bolts are not properly torqued. Thus, B' is the event that all bolts are properly torqued. Then,

$$P(B) = 1 - P(B') = 1 - \left(\frac{15}{20}\right)\left(\frac{14}{19}\right)\left(\frac{13}{18}\right)\left(\frac{12}{17}\right) = 0.718$$

2-113. D = defective copy

a) $P(D = 1) = \left(\frac{2}{75}\right)\left(\frac{73}{74}\right)\left(\frac{72}{73}\right) + \left(\frac{73}{75}\right)\left(\frac{2}{74}\right)\left(\frac{72}{73}\right) + \left(\frac{73}{75}\right)\left(\frac{72}{74}\right)\left(\frac{2}{73}\right) = 0.0778$

b) $P(D = 2) = \left(\frac{2}{75}\right)\left(\frac{1}{74}\right)\left(\frac{73}{73}\right) + \left(\frac{2}{75}\right)\left(\frac{73}{74}\right)\left(\frac{1}{73}\right) + \left(\frac{73}{75}\right)\left(\frac{2}{74}\right)\left(\frac{1}{73}\right) = 0.00108$

2-117. Let A_i denote the event that the ith washer selected is thicker than target.

a) $\left(\frac{30}{50}\right)\left(\frac{29}{49}\right)\left(\frac{28}{8}\right) = 0.207$

b) 30/48 = 0.625

c) The requested probability can be written in terms of whether or not the first and second washer selected are thicker than the target. That is,

$$P(A_3) = P(A_1A_2A_3 \text{ or } A_1A_2'A_3 \text{ or } A_1'A_2A_3 \text{ or } A_1'A_2'A_3)$$
$$= P(A_3 | A_1A_2)P(A_1A_2) + P(A_3 | A_1A_2')P(A_1A_2')$$
$$+ P(A_3 | A_1'A_2)P(A_1'A_2) + P(A_3 | A_1'A_2')P(A_1'A_2')$$
$$= P(A_3 | A_1A_2)P(A_2 | A_1)P(A_1) + P(A_3 | A_1A_2')P(A_2' | A_1)P(A_1)$$
$$+ P(A_3 | A_1'A_2)P(A_2 | A_1')P(A_1') + P(A_3 | A_1'A_2')P(A_2' | A_1')P(A_1')$$
$$= \frac{28}{48}\left(\frac{30}{50}\frac{29}{49}\right) + \frac{29}{48}\left(\frac{20}{50}\frac{30}{49}\right) + \frac{29}{48}\left(\frac{20}{50}\frac{30}{49}\right) + \frac{30}{48}\left(\frac{20}{50}\frac{19}{49}\right)$$
$$= 0.60$$

2-121. Let A_i denote the event that the ith row operates. Then,

$P(A_1) = 0.98, P(A_2) = (0.99)(0.99) = 0.9801, P(A_3) = 0.9801, \ P(A_4) = 0.98.$

The probability the circuit does not operate is

$$P(A_1')P(A_2')P(A_3')P(A_4') = (0.02)(0.0199)(0.0199)(0.02) = 1.58 \times 10^{-7}$$