Chapter 3 Selected Problem Solutions

Section 3-2

3-13.

$$f_X(0) = P(X = 0) = 1/6 + 1/6 = 1/3$$
$$f_X(1.5) = P(X = 1.5) = 1/3$$
$$f_X(2) = 1/6$$
$$f_X(3) = 1/6$$

3-21. $P(X = 0) = 0.02^3 = 8 \times 10^{-6}$
$P(X = 1) = 3[0.98(0.02)(0.02)]=0.0012$
$P(X = 2) = 3[0.98(0.98)(0.02)]=0.0576$
$P(X = 3) = 0.98^3 = 0.9412$

3-25. X = number of components that meet specifications
P(X=0) = (0.05)(0.02)(0.01) = 0.00001
P(X=1) = (0.95)(0.02)(0.01) + (0.05)(0.98)(0.01)+(0.05)(0.02)(0.99) = 0.00167
P(X=2) = (0.95)(0.98)(0.01) + (0.95)(0.02)(0.99) + (0.05)(0.98)(0.99) = 0.07663
P(X=3) = (0.95)(0.98)(0.99) = 0.92169

Section 3-3

3-27.

$$F(x) = \begin{cases} 0, & x < -2 \\ 1/8 & -2 \le x < -1 \\ 3/8 & -1 \le x < 0 \\ 5/8 & 0 \le x < 1 \\ 7/8 & 1 \le x < 2 \\ 1 & 2 \le x \end{cases}$$

a) $P(X \le 1.25) = 7/8$
b) $P(X \le 2.2) = 1$
c) $P(-1.1 < X \le 1) = 7/8 - 1/8 = 3/4$
d) $P(X > 0) = 1 - P(X \le 0) = 1 - 5/8 = 3/8$

3-31.

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.008 & 0 \le x < 1 \\ 0.104 & 1 \le x < 2 \\ 0.488 & 2 \le x < 3 \\ 1 & 3 \le x \end{cases} \text{ where } \begin{aligned} f(0) &= 0.2^3 = 0.008, \\ f(1) &= 3(0.2)(0.2)(0.8) = 0.096, \\ f(2) &= 3(0.2)(0.8)(0.8) = 0.384, \\ f(3) &= (0.8)^3 = 0.512, \end{aligned}$$

3-33. a) $P(X \le 3) = 1$
b) $P(X \le 2) = 0.5$
c) $P(1 \le X \le 2) = P(X=1) = 0.5$
d) $P(X>2) = 1 - P(X\le 2) = 0.5$

Section 3-4

3-37 Mean and Variance

$$\mu = E(X) = 0f(0) + 1f(1) + 2f(2) + 3f(3) + 4f(4)$$
$$= 0(0.2) + 1(0.2) + 2(0.2) + 3(0.2) + 4(0.2) = 2$$
$$V(X) = 0^2 f(0) + 1^2 f(1) + 2^2 f(2) + 3^2 f(3) + 4^2 f(4) - \mu^2$$
$$= 0(0.2) + 1(0.2) + 4(0.2) + 9(0.2) + 16(0.2) - 2^2 = 2$$

3-41. Mean and variance for exercise 3-19

$$\mu = E(X) = 10f(10) + 5f(5) + 1f(1)$$
$$= 10(0.3) + 5(0.6) + 1(0.1)$$
$$= 6.1 \text{ million}$$
$$V(X) = 10^2 f(10) + 5^2 f(5) + 1^2 f(1) - \mu^2$$
$$= 10^2(0.3) + 5^2(0.6) + 1^2(0.1) - 6.1^2$$
$$= 7.89 \text{ million}^2$$

3-45. Determine x where range is [0,1,2,3,x] and mean is 6.

$$\mu = E(X) = 6 = 0f(0) + 1f(1) + 2f(2) + 3f(3) + xf(x)$$
$$6 = 0(0.2) + 1(0.2) + 2(0.2) + 3(0.2) + x(0.2)$$
$$6 = 1.2 + 0.2x$$
$$4.8 = 0.2x$$
$$x = 24$$

Section 3-5

3-47. E(X) = (3+1)/2 = 2, V(X) = $[(3-1+1)^2 -1]/12$ = 0.667

3-49. X=(1/100)Y, Y = 15, 16, 17, 18, 19.

E(X) = (1/100) E(Y) = $\frac{1}{100}\left(\frac{15+19}{2}\right) = 0.17$ mm

$$V(X) = \left(\frac{1}{100}\right)^2 \left[\frac{(19-15+1)^2 - 1}{12}\right] = 0.0002 \text{ mm}^2$$

Section 3-6

3-57. a) $P(X = 5) = \binom{10}{5} 0.5^5 (0.5)^5 = 0.2461$

b) $P(X \le 2) = \binom{10}{0} 0.5^0 0.5^{10} + \binom{10}{1} 0.5^1 0.5^9 + \binom{10}{2} 0.5^2 0.5^8$

$$= 0.5^{10} + 10(0.5)^{10} + 45(0.5)^{10} = 0.0547$$

c) $P(X \ge 9) = \binom{10}{9} 0.5^9 (0.5)^1 + \binom{10}{10} 0.5^{10} (0.5)^0 = 0.0107$

d) $P(3 \le X < 5) = \binom{10}{3} 0.5^3 0.5^7 + \binom{10}{4} 0.5^4 0.5^6$

$$= 120(0.5)^{10} + 210(0.5)^{10} = 0.3223$$

3-61. n=3 and p=0.25

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.4219 & 0 \le x < 1 \\ 0.8438 & 1 \le x < 2 \\ 0.9844 & 2 \le x < 3 \\ 1 & 3 \le x \end{cases} \text{ where } \begin{array}{l} f(0) = \left(\frac{3}{4}\right)^3 = \frac{27}{64} \\ f(1) = 3\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^2 = \frac{27}{64} \\ f(2) = 3\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right) = \frac{9}{64} \\ f(3) = \left(\frac{1}{4}\right)^3 = \frac{1}{64} \end{array}$$

3-63. a) $P(X = 1) = \binom{1000}{1} 0.001^1 (0.999)^{999} = 0.3681$

$b) P(X \ge 1) = 1 - P(X = 0) = 1 - \binom{1000}{1} 0.001^1 (0.999)^{999} = 0.6323$

$c) P(X \le 2) = \binom{1000}{0} 0.001^0 (0.999)^{1000} + \binom{1000}{1} 0.001^1 (0.999)^{999} + \binom{1000}{2} 0.001^2 0.999^{998}$

$$= 0.9198$$

$d) E(X) = 1000(0.001) = 1$

$V(X) = 1000(0.001)(0.999) = 0.999$

3-67. Let X denote the passengers with tickets that do not show up for the flight. Then, X is binomial with n = 125 and p = 0.1.

$a) P(X \ge 5) = 1 - P(X \le 4)$

$$= 1 - \left[\binom{125}{0} 0.1^0 (0.9)^{125} + \binom{125}{1} 0.1^1 (0.9)^{124} + \binom{125}{2} 0.1^2 (0.9)^{123} + \binom{125}{3} 0.1^3 (0.9)^{122} + \binom{125}{4} 0.1^4 (0.9)^{121} \right] = 0.9961$$

$b) P(X > 5) = 1 - P(X \le 5) = 0.9886$

3-69. Let X denote the number of questions answered correctly. Then, X is binomial with n = 25 and p = 0.25.

$$a) P(X \geq 20) = \binom{25}{20} 0.25^{20}(0.75)^{5} + \binom{25}{21} 0.25^{21}(0.75)^{4} + \binom{25}{22} 0.25^{22}(0.75)^{3}$$
$$+ \binom{25}{23} 0.25^{23}(0.75)^{2} + \binom{25}{24} 0.25^{24}(0.75)^{1} + \binom{25}{25} 0.25^{25}(0.75)^{0} \cong 0$$

$$b) P(X < 5) = \binom{25}{0} 0.25^{0}(0.75)^{25} + \binom{25}{1} 0.25^{1}(0.75)^{24} + \binom{25}{2} 0.25^{2}(0.75)^{23}$$
$$+ \binom{25}{3} 0.25^{3}(0.75)^{22} + \binom{25}{4} 0.25^{4}(0.75)^{21} = 0.2137$$

Section 3-7

3-71. a. $P(X = 1) = (1-0.5)^{0} 0.5 = 0.5$

b. $P(X = 4) = (1-0.5)^{3} 0.5 = 0.5^{4} = 0.0625$

c. $P(X = 8) = (1-0.5)^{7} 0.5 = 0.5^{8} = 0.0039$

d. $P(X \leq 2) = P(X = 1) + P(X = 2) = (1-0.5)^{0} 0.5 + (1-0.5)^{1} 0.5$

$= 0.5 + 0.5^{2} = 0.75$

e. $P(X > 2) = 1 - P(X \leq 2) = 1 - 0.75 = 0.25$

3-75. Let X denote the number of calls needed to obtain a connection. Then, X is a geometric random variable with p = 0.02

a) $P(X = 10) = (1-0.02)^{9} 0.02 = 0.98^{9} 0.02 = 0.0167$

b) $P(X > 5) = 1 - P(X \leq 4) = 1 - [P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)]$

$= 1 - [0.02 + 0.98(0.02) + 0.98^{2}(0.02) + 0.98^{3}(0.02)]$

$= 1 - 0.0776 = 0.9224$

c) E(X) = 1/0.02 = 50

3-77 p = 0.005 , r = 8

a. $P(X = 8) = 0.0005^{8} = 3.91x10^{-19}$

b. $\mu = E(X) = \dfrac{1}{0.005} = 200$ days

c Mean number of days until all 8 computers fail. Now we use p=$3.91x10^{-19}$

$\mu = E(Y) = \dfrac{1}{3.91x10^{-91}} = 2.56x10^{18}$ days or 7.01 $x10^{15}$ years

3-81. a) E(X) = 4/0.2 = 20

b) P(X=20) = $\binom{19}{3}(0.80)^{16} 0.2^{4} = 0.0436$

c) P(X=19) = $\binom{18}{3}(0.80)^{15} 0.2^{4} = 0.0459$

d) P(X=21) = $\binom{20}{3}(0.80)^{17} 0.2^{4} = 0.0411$

e) The most likely value for X should be near μ_X. By trying several cases, the most likely value is x = 19.

3-83. Let X denote the number of fills needed to detect three underweight packages. Then X is a negative binomial random variable with p = 0.001 and r = 3.

a) E(X) = 3/0.001 = 3000

b) V(X) = [3(0.999)/0.001^2] = 2997000. Therefore, σ_X = 1731.18

Section 3-8

3-87. a) $$P(X=1)=\frac{\binom{4}{1}\binom{16}{3}}{\binom{20}{4}}=\frac{(4\times16\times15\times14)/6}{(20\times19\times18\times17)/24}=0.4623$$

b) $$P(X=4)=\frac{\binom{4}{4}\binom{16}{0}}{\binom{20}{4}}=\frac{1}{(20\times19\times18\times17)/24}=0.00021$$

c)
$$P(X\le2)=P(X=0)+P(X=1)+P(X=2)$$
$$=\frac{\binom{4}{0}\binom{16}{4}}{\binom{20}{4}}+\frac{\binom{4}{1}\binom{16}{3}}{\binom{20}{4}}+\frac{\binom{4}{2}\binom{16}{2}}{\binom{20}{4}}$$
$$=\frac{\left(\frac{16\times15\times14\times13}{24}+\frac{4\times16\times15\times14}{6}+\frac{6\times16\times15}{2}\right)}{\left(\frac{20\times19\times18\times17}{24}\right)}=0.9866$$

d) E(X) = 4(4/20) = 0.8
V(X) = 4(0.2)(0.8)(16/19) = 0.539

3-91. Let X denote the number of men who carry the marker on the male chromosome for an increased risk for high blood pressure. N=800, K=240 n=10

a) n=10
$$P(X=1)=\frac{\binom{240}{1}\binom{560}{9}}{\binom{800}{10}}=\frac{\left(\frac{240!}{1!239!}\right)\left(\frac{560!}{9!551!}\right)}{\frac{800!}{10!790!}}=0.1201$$

b) n=10
$$P(X>1)=1-P(X\le1)=1-[P(X=0)+P(X=1)]$$
$$P(X=0)=\frac{\binom{240}{0}\binom{560}{10}}{\binom{800}{10}}=\frac{\left(\frac{240!}{0!240!}\right)\left(\frac{560!}{10!560!}\right)}{\frac{800!}{10!790!}}=0.0276$$
$$P(X>1)=1-P(X\le1)=1-[0.0276+0.1201]=0.8523$$

Section 3-9

3-97. a) $$P(X=0)=\frac{e^{-4}4^0}{0!}=e^{-4}=0.0183$$

b) $P(X\le2)=P(X=0)+P(X=1)+P(X=2)$
$$=e^{-4}+\frac{e^{-4}4^1}{1!}+\frac{e^{-4}4^2}{2!}=0.2381$$

c) $$P(X=4)=\frac{e^{-4}4^4}{4!}=0.1954$$

d) $P(X = 8) = \frac{e^{-4} 4^8}{8!} = 0.0298$

3-99. $P(X = 0) = e^{-\lambda} = 0.05$. Therefore, $\lambda = -\ln(0.05) = 2.996$.
Consequently, E(X) = V(X) = 2.996.

3-101. a) Let X denote the number of flaws in one square meter of cloth. Then, X is a Poisson random variable with $\lambda = 0.1$. $P(X = 2) = \frac{e^{-0.1}(0.1)^2}{2!} = 0.0045$

b) Let Y denote the number of flaws in 10 square meters of cloth. Then, Y is a Poisson random variable with $\lambda = 1$. $P(Y = 1) = \frac{e^{-1}1^1}{1!} = e^{-1} = 0.3679$

c) Let W denote the number of flaws in 20 square meters of cloth. Then, W is a Poisson random variable with $\lambda = 2$. $P(W = 0) = e^{-2} = 0.1353$

d)
$$\begin{aligned} P(Y \geq 2) &= 1 - P(Y \leq 1) = 1 - P(Y = 0) - P(Y = 1) \\ &= 1 - e^{-1} - e^{-1} \\ &= 0.2642 \end{aligned}$$

3-105. a) Let X denote the number of flaws in 10 square feet of plastic panel. Then, X is a Poisson random variable with $\lambda = 0.5$. $P(X = 0) = e^{-0.5} = 0.6065$

b) Let Y denote the number of cars with no flaws,

$$P(Y = 10) = \binom{10}{10}(0.3935)^{10}(0.6065)^0 = 8.9x10^{-5}$$

c) Let W denote the number of cars with surface flaws. Because the number of flaws has a Poisson distribution, the occurrences of surface flaws in cars are independent events with constant probability. From part a., the probability a car contains surface flaws is 1–0.6065 = 0.3935. Consequently, W is binomial with n = 10 and p = 0.3935.

$$P(W = 0) = \binom{10}{0}(0.6065)^0(0.3935)^{10} = 8.9x10^{-5}$$

$$P(W = 1) = \binom{10}{1}(0.6065)^1(0.3935)^9 = 0.001372$$

$$P(W \leq 1) = 0.000089 + 0.001372 = 0.00146$$

Supplemental Exercises

3-107. Let X denote the number of totes in the sample that do not conform to purity requirements. Then, X has a hypergeometric distribution with N = 15, n = 3, and K = 2.

$$P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{\binom{2}{0}\binom{13}{3}}{\binom{15}{3}} = 1 - \frac{13!12!}{10!15!} = 0.3714$$

3-109. Let Y denote the number of calls needed to obtain an answer in less than 30 seconds.

a) $P(Y = 4) = (1 - 0.75)^3 0.75 = 0.25^3 0.75 = 0.0117$

b) E(Y) = 1/p = 1/0.75 = 1.3333

3-111. a) Let X denote the number of messages sent in one hour. $P(X = 5) = \frac{e^{-5} 5^5}{5!} = 0.1755$

b) Let Y denote the number of messages sent in 1.5 hours. Then, Y is a Poisson random variable with λ =7.5. $P(Y = 10) = \frac{e^{-7.5} (7.5)^{10}}{10!} = 0.0858$

c) Let W denote the number of messages sent in one-half hour. Then, W is a Poisson random variable with λ = 2.5. $P(W < 2) = P(W = 0) + P(W = 1) = 0.2873$

3-119. Let X denote the number of products that fail during the warranty period. Assume the units are independent. Then, X is a binomial random variable with n = 500 and p = 0.02.

a) P(X = 0) = $\binom{500}{0}(0.02)^0 (0.98)^{500} = 4.1 \times 10^{-5}$

b) E(X) = 500(0.02) = 10

c) P(X >2) = 1 – P(X ≤ 1) = 0.9995

3-121. a) P(X ≤ 3) = 0.2 + 0.4 = 0.6

b) P(X > 2.5) = 0.4 + 0.3 + 0.1 = 0.8

c) P(2.7 < X < 5.1) = 0.4 + 0.3 = 0.7

d) E(X) = 2(0.2) + 3(0.4) + 5(0.3) + 8(0.1) = 3.9

e) V(X) = $2^2(0.2) + 3^2(0.4) + 5^2(0.3) + 8^2(0.1) - (3.9)^2 = 3.09$

3-125. Let X denote the number of orders placed in a week in a city of 800,000 people. Then X is a Poisson random variable with λ = 0.25(8) = 2.

a) P(X ≥ 3) = 1 – P(X ≤ 2) = $1 - [e^{-2} + e^{-2}(2) + (e^{-2}2^2)/2!] = 1 - 0.6767 = 0.3233$.

b) Let Y denote the number of orders in 2 weeks. Then, Y is a Poisson random variable with λ = 4, and $P(Y<2) = P(Y \le 1) = e^{-4} + (e^{-4}4^1)/1! = 0.0916$.

3-127. Let X denote the number of totes in the sample that exceed the moisture content. Then X is a binomial random variable with n = 30. We are to determine p.

If P(X ≥ 1) = 0.9, then P(X = 0) = 0.1. Then $\binom{30}{0}(p)^0 (1-p)^{30} = 0.1$, giving 30ln(1–p)=ln(0.1),

which results in p = 0.0738.

3-129. a) Let X denote the number of flaws in 50 panels. Then, X is a Poisson random variable with λ = 50(0.02) = 1. P(X = 0) = e^{-1} = 0.3679.

b) Let Y denote the number of flaws in one panel, then $P(Y \ge 1) = 1 - P(Y=0) = 1 - e^{-0.02} = 0.0198$. Let W denote the number of panels that need to be inspected before a flaw is found. Then W is a geometric random variable with p = 0.0198 and E(W) = 1/0.0198 = 50.51 panels.

c.) $P(Y \ge 1) = 1 - P(Y = 0) = 1 - e^{-0.02} = 0.0198$

Let V denote the number of panels with 2 or more flaws. Then V is a binomial random variable with n=50 and p=0.0198

$$P(V \le 2) = \binom{50}{0} 0.0198^{0} (.9802)^{50} + \binom{50}{1} 0.0198^{1} (0.9802)^{49}$$

$$+ \binom{50}{2} 0.0198^{2} (0.9802)^{48} = 0.9234$$