**Chapter 5 Selected Problem Solutions**

Section 5-1

5-7.

$$E(X) = 1[f_{XY}(1,1) + f_{XY}(1,2) + f_{XY}(1,3)] + 2[f_{XY}(2,1) + f_{XY}(2,2) + f_{XY}(2,3)]$$
$$+ \ 3[f_{XY}(3,1) + f_{XY}(3,2) + f_{XY}(3,3)]$$
$$= (1\times\tfrac{9}{36}) + (2\times\tfrac{12}{36}) + (3\times\tfrac{15}{36}) = 13/6 = 2.167$$
$$V(X) = (1-\tfrac{13}{6})^2\ \tfrac{9}{36} + (2-\tfrac{13}{6})^2\ \tfrac{12}{36} + (3-\tfrac{13}{6})^2\ \tfrac{15}{36} = 0.639$$
$$E(Y) = 2.167$$
$$V(Y) = 0.639$$

5-11.

$$E(X) = -1(\tfrac{1}{8}) - 0.5(\tfrac{1}{4}) + 0.5(\tfrac{1}{2}) + 1(\tfrac{1}{8}) = \tfrac{1}{8}$$
$$E(Y) = -2(\tfrac{1}{8}) - 1(\tfrac{1}{4}) + 1(\tfrac{1}{2}) + 2(\tfrac{1}{8}) = \tfrac{1}{4}$$

5-15 a) The range of (X,Y) is $X \ge 0,\ Y \ge 0\ and\ X + Y \le 4$ . X is the number of pages with moderate graphic content and Y is the number of pages with high graphic output out of 4.

| | x=0 | x=1 | x=2 | x=3 | x=4 |
|---|---|---|---|---|---|
| y=4 | 5.35x10$^{-05}$ | 0 | 0 | 0 | 0 |
| y=3 | 0.00183 | 0.00092 | 0 | 0 | 0 |
| y=2 | 0.02033 | 0.02066 | 0.00499 | 0 | 0 |
| y=1 | 0.08727 | 0.13542 | 0.06656 | 0.01035 | 0 |
| y=0 | 0.12436 | 0.26181 | 0.19635 | 0.06212 | 0.00699 |

b.)

| | x=0 | x=1 | x=2 | x=3 | x=4 |
|---|---|---|---|---|---|
| f(x) | 0.2338 | 0.4188 | 0.2679 | 0.0725 | 0.0070 |

c.)
E(X)=

$$\sum_{0}^{4} x_i f(x_i) = 0(0.2338) + 1(0.4188) + 2(0.2679) + 3(0.7248) = 4(0.0070) = 1.2$$

d.) $f_{Y|3}(y) = \dfrac{f_{XY}(3,y)}{f_X(3)}$ , $f_x(3) = 0.0725$

| y | $f_{Y\|3}(y)$ |
|---|---|
| 0 | 0.857 |
| 1 | 0.143 |
| 2 | 0 |
| 3 | 0 |
| 4 | 0 |

e) E(Y|X=3) = 0(0.857)+1(0.143) = 0.143

Section 5-2

5-17. a) $P(X=2)=f_{XYZ}(2,1,1)+f_{XYZ}(2,1,2)+f_{XYZ}(2,2,1)+f_{XYZ}(2,2,2)=0.5$

b) $P(X=1,Y=2)=f_{XYZ}(1,2,1)+f_{XYZ}(1,2,2)=0.35$

c) $P(Z<1.5)=f_{XYZ}(1,1,1)+f_{XYZ}(1,2,2)+f_{XYZ}(2,1,1)+f_{XYZ}(2,2,1)=0.5$

d)

$$P(X=1 \ \ or \ \ Z=1)=P(X=1)+P(Z=1)-P(X=1,Z=1)=0.5+0.5-0.2=0.8$$

e) E(X) = 1(0.5) + 2(0.5) = 1.5

5-25. P(X=x, Y=y, Z=z) is the number of subsets of size 4 that contain x printers with graphics enhancements, y printers with extra memory, and z printers with both features divided by the number of subsets of size 4. From the results on the CD material on counting techniques, it can be shown that

$$P(X=x,Y=y,Z=z)=\frac{\binom{4}{x}\binom{5}{y}\binom{6}{z}}{\binom{15}{4}} \quad \text{for } x+y+z=4.$$

a) $P(X=1,Y=2,Z=1)=\dfrac{\binom{4}{1}\binom{5}{2}\binom{6}{1}}{\binom{15}{4}}=0.1758$

b) $P(X=1,Y=1)=P(X=1,Y=1,Z=2)=\dfrac{\binom{4}{1}\binom{5}{1}\binom{6}{2}}{\binom{15}{4}}=0.2198$

c) The marginal distribution of X is hypergeometric with N = 15, n = 4, K = 4.
Therefore, E(X) = nK/N = 16/15 and V(X) = 4(4/15)(11/15)[11/14] = 0.6146.

5-29 a.) $P(X=2\mid Y=2)=\dfrac{P(X=2,Y=2)}{P(Y=2)}=\dfrac{0.1944}{0.2646}=0.7347$

$$P(X=2,Y=2)=0.1922$$

$$P(Y=2)=\binom{4}{2}0.3^2 0.7^4=0.2646 \quad \text{from the binomial marginal distribution of } Y$$

b) Not possible, $x+y+z=4$, the probability is zero.

c.) $P(X\mid Y=2)=P(X=0\mid Y=2),P(X=1\mid Y=2),P(X=2\mid Y=2)$

$$P(X=0\mid Y=2)=\frac{P(X=0,Y=2)}{P(Y=2)}=\left(\frac{4!}{0!2!2!}0.6^0 0.3^2 0.1^2\right)\Big/0.2646=0.0204$$

$$P(X=1\mid Y=2)=\frac{P(X=1,Y=2)}{P(Y=2)}=\left(\frac{4!}{1!2!1!}0.6^1 0.3^2 0.1^1\right)\Big/0.2646=0.2449$$

$$P(X=2\mid Y=2)=\frac{P(X=2,Y=2)}{P(Y=2)}=\left(\frac{4!}{2!2!0!}0.6^2 0.3^2 0.1^0\right)\Big/0.2646=0.7347$$

d.) E(X|Y=2)=0(0.0204)+1(0.2449)+2(0.7347) = 1.7142

5-31 a.), X has a binomial distribution with n = 3 and p = 0.01. Then, E(X) = 3(0.01) = 0.03 and V(X) = 3(0.01)(0.99) = 0.0297.

b.) first find $P(X \mid Y = 2)$

$$P(Y = 2) = P(X = 1, Y = 2, Z = 0) + P(X = 0, Y = 2, Z = 1)$$

$$= \frac{3!}{1!2!0!} 0.01(0.04)^2 0.95^0 + \frac{3!}{0!2!1!} 0.01^0 (0.04)^2 0.95^1 = 0.0046$$

$$P(X = 0 \mid Y = 2) = \frac{P(X = 0, Y = 2)}{P(Y = 2)} = \left( \frac{3!}{0!2!1!} 0.01^0 0.04^2 0.95^1 \right) \Big/ 0.004608 = 0.98958$$

$$P(X = 1 \mid Y = 2) = \frac{P(X = 1, Y = 2)}{P(Y = 2)} = \left( \frac{3!}{1!2!1!} 0.01^1 0.04^2 0.95^0 \right) \Big/ 0.004608 = 0.01042$$

$$E(X \mid Y = 2) = 0(0.98958) + 1(0.01042) = 0.01042$$

$$V(X \mid Y = 2) = E(X^2) - (E(X))^2 = 0.01042 - (0.01042)^2 = 0.01031$$

Section 5-3

5-35. a) $P(X < 2, Y < 3) = \frac{4}{81} \int_0^3 \int_0^2 xydxdy = \frac{4}{81}(2) \int_0^3 ydy = \frac{4}{81}(2)(\frac{9}{2}) = 0.4444$

b) P(X < 2.5) = P(X < 2.5, Y < 3) because the range of Y is from 0 to 3.

$$P(X < 2.5, Y < 3) = \frac{4}{81} \int_0^3 \int_0^{2.5} xydxdy = \frac{4}{81}(3.125) \int_0^3 ydy = \frac{4}{81}(3.125)\frac{9}{2} = 0.6944$$

c) $P(1 < Y < 2.5) = \frac{4}{81} \int_1^{2.5} \int_0^3 xydxdy = \frac{4}{81}(4.5) \int_1^{2.5} ydy = \frac{18}{81} \frac{y^2}{2} \Big|_1^{2.5} = 0.5833$

d) $P(X > 1.8, 1 < Y < 2.5) = \frac{4}{81} \int_1^{2.5} \int_{1.8}^3 xydxdy = \frac{4}{81}(2.88) \int_1^{2.5} ydy = \frac{4}{81}(2.88)\frac{(2.5^2 - 1)}{2} = 0.3733$

e) $E(X) = \frac{4}{81} \int_0^3 \int_0^3 x^2 ydxdy = \frac{4}{81} \int_0^3 9ydy = \frac{4}{9} \frac{y^2}{2} \Big|_0^3 = 2$

f) $P(X < 0, Y < 4) = \frac{4}{81} \int_0^4 \int_0^0 xydxdy = 0 \int_0^4 ydy = 0$

5-37.

$$c\int_0^3\int_x^{x+2}(x+y)\,dy\,dx = \int_0^3 xy + \frac{y^2}{2}\Big|_x^{x+2}\,dx$$

$$= \int_0^3 \left[x(x+2) + \frac{(x+2)^2}{2} - x^2 - \frac{x^2}{2}\right]dx$$

$$= c\int_0^3 (4x+2)\,dx = \left[2x^2 + 2x\right]_0^3 = 24c$$

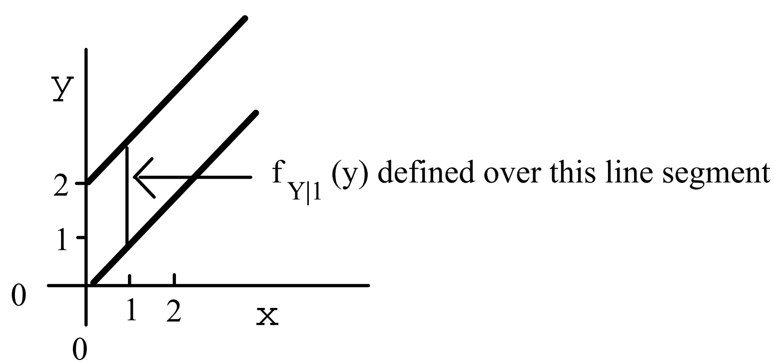
Therefore, c = 1/24.

5-39. a) $f_X(x)$ is the integral of $f_{XY}(x,y)$ over the interval from x to x+2. That is,

$$f_X(x) = \frac{1}{24}\int_x^{x+2}(x+y)\,dy = \frac{1}{24}\left[xy + \frac{y^2}{2}\Big|_x^{x+2}\right] = \frac{x}{6} + \frac{1}{12} \quad \text{for } 0 < x < 3.$$

b) $f_{Y|1}(y) = \frac{f_{XY}(1,y)}{f_X(1)} = \frac{\frac{1}{24}(1+y)}{\frac{1}{6}+\frac{1}{12}} = \frac{1+y}{6}$ for $1 < y < 3$.

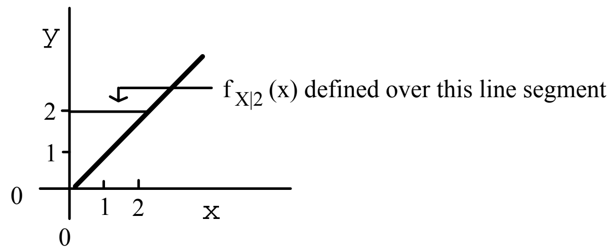
See the following graph,




c) $E(Y|X=1) = \int_1^3 y\left(\frac{1+y}{6}\right)dy = \frac{1}{6}\int_1^3 (y+y^2)\,dy = \frac{1}{6}\left(\frac{y^2}{2}+\frac{y^3}{3}\right)\Big|_1^3 = 2.111$

d.) $P(Y > 2 \mid X = 1) = \int_2^3 \left(\frac{1+y}{6}\right)dy = \frac{1}{6}\int_1^3 (1+y)\,dy = \frac{1}{6}\left(y + \frac{y^2}{2}\right)\Big|_1^2 = 0.4167$

e.) $f_{X|2}(x) = \frac{f_{XY}(x,2)}{f_Y(2)}$ . Here $f_Y(y)$ is determined by integrating over *x*. There are three regions of integration. For $0 < y \le 2$ the integration is from 0 to *y*. For $2 < y \le 3$ the integration is from *y*-2 to *y*. For $3 < y < 5$ the integration is from *y* to 3. Because the condition is *x*=2, only the first integration is needed. $f_Y(y) = \frac{1}{24}\int_0^y (x+y)\,dx = \frac{1}{24}\left[\frac{x^2}{2} + xy\Big|_0^y\right] = \frac{y^2}{16}$ for $0 < y \le 2$.



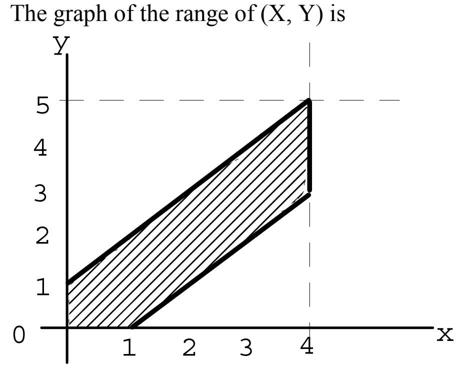

Therefore, $f_Y(2) = 1/4$ and $f_{X|2}(x) = \dfrac{\frac{1}{24}(x+2)}{1/4} = \dfrac{x+2}{6}$ for 0 < x < 2

5-43. Solve for c

$$c\int_0^{\infty}\int_0^{x} e^{-2x-3y}\,dydx = \frac{c}{3}\int_0^{\infty} e^{-2x}\left(1-e^{-3x}\right)dx = \frac{c}{3}\int_0^{\infty} e^{-2x} - e^{-5x}\,dx$$

$$= \frac{c}{3}\left(\frac{1}{2}-\frac{1}{5}\right) = \frac{1}{10}c. \quad c = 10$$

5-49. The graph of the range of (X, Y) is




$$\int_0^1\int_0^{x+1} cdydx + \int_1^4\int_{x-1}^{x+1} cdydx = 1$$

$$= c\int_0^1 (x+1)dx + 2c\int_1^4 dx$$

$$= \tfrac{3}{2}c + 6c = 7.5c = 1$$

Therefore, $c$ = 1/7.5=2/15

5-51. a. )

$$f(x) = \int_0^{x+1} \frac{1}{7.5}\,dy = \left(\frac{x+1}{7.5}\right) \quad \text{for} \quad 0 < x < 1,$$

$$f(x) = \int_{x-1}^{x+1} \frac{1}{7.5}\,dy = \left(\frac{x+1-(x-1)}{7.5}\right) = \frac{2}{7.5} \quad \text{for} \quad 1 < x < 4$$

b. )

$$f_{Y|X=1}(y) = \frac{f_{XY}(1,y)}{f_X(1)} = \frac{1/7.5}{2/7.5} = 0.5$$

$$f_{Y|X=1}(y) = 0.5 \quad \text{for } 0 < y < 2$$

c. ) $E(Y \mid X = 1) = \int_0^2 \frac{y}{2} dy = \left.\frac{y^2}{4}\right|_0^2 = 1$

d.) $P(Y < 0.5 \mid X = 1) = \int_0^{0.5} 0.5 dy = 0.5y\Big|_0^{0.5} = 0.25$

5-53 a.) $\mu$=3.2 $\lambda$=1/3.2

$$P(X > 5, Y > 5) = 10.24 \int_5^\infty \int_5^\infty e^{-\frac{x}{3.2}-\frac{y}{3.2}} dydx = 3.2 \int_5^\infty e^{-\frac{x}{3.2}} \left( e^{-\frac{5}{3.2}} \right) dx$$

$$= \left( e^{-\frac{5}{3.2}} \right)\left( e^{-\frac{5}{3.2}} \right) = 0.0439$$

$$P(X > 10, Y > 10) = 10.24 \int_{10}^\infty \int_{10}^\infty e^{-\frac{x}{3.2}-\frac{y}{3.2}} dydx = 3.2 \int_{10}^\infty e^{-\frac{x}{3.2}} \left( e^{-\frac{10}{3.2}} \right) dx$$

$$= \left( e^{-\frac{10}{3.2}} \right)\left( e^{-\frac{10}{3.2}} \right) = 0.0019$$

b.) Let *X* denote the number of orders in a 5-minute interval. Then *X* is a Poisson random variable with $\lambda$=5/3.2 = 1.5625.

$$P(X = 2) = \frac{e^{-1.5625}(1.5625)^2}{21} = 0.256$$

For both systems, $P(X = 2)P(X = 2) = 0.256^2 = 0.0655$

c.) The joint probability distribution is not necessary because the two processes are independent and we can just multiply the probabilities.

Section 5-4

5-55. a) $P(X < 0.5) = \int_0^{0.5} \int_0^1 \int_0^1 (8xyz)dzdydx = \int_0^{0.5} \int_0^1 (4xy)dydx = \int_0^{0.5} (2x)dx = x^2 \Big|_0^{0.5} = 0.25$

b)

$$P(X < 0.5, Y < 0.5) = \int_0^{0.5}\int_0^{0.5}\int_0^{1} (8xyz)dzdydx$$

$$= \int_0^{0.5}\int_0^{0.5} (4xy)dydx = \int_0^{0.5} (0.5x)dx = \left.\tfrac{x^2}{4}\right|_0^{0.5} = 0.0625$$

c) P(Z < 2) = 1, because the range of Z is from 0 to 1.
d) P(X < 0.5 or Z < 2) = P(X < 0.5) + P(Z < 2) - P(X < 0.5, Z < 2). Now, P(Z < 2) =1 and
P(X < 0.5, Z < 2) = P(X < 0.5). Therefore, the answer is 1.

e) $E(X) = \int_0^1\int_0^1\int_0^1 (8x^2 yz)dzdydx = \int_0^1 (2x^2)dx = \tfrac{2x^3}{3} = 2/3$

5-57. a) $f_{YZ}(y,z) = \int_0^1 (8xyz)dx = 4yz$ for 0 < y < 1 and 0 < z < 1.

Then, $f_{X|YZ}(x) = \dfrac{f_{XYZ}(x,y,z)}{f_{YZ}(y,z)} = \dfrac{8x(0.5)(0.8)}{4(0.5)(0.8)} = 2x$ for 0 < x < 1.

b) Therefore, $P(X < 0.5 | Y = 0.5, Z = 0.8) = \int_0^{0.5} 2xdx = 0.25$

5-61 Determine c such that $f(xyz) = c$ is a joint density probability over the region x>0, y>0 and z>0 with x+y+z<1

$$f(xyz) = c\int_0^1\int_0^{1-x}\int_0^{1-x-y} dzdydx = \int_0^1\int_0^{1-x} c(1-x-y)dydx = \int_0^1 \left( c(y - xy - \frac{y^2}{2})\Big|_0^{1-x} \right) dx$$

$$= \int_0^1 c\left( (1-x) - x(1-x) - \frac{(1-x)^2}{2} \right) dx = \int_0^1 c\left( \frac{(1-x)^2}{2} \right) dx = c\left( \frac{1}{2}x - \frac{x^2}{2} + \frac{x^3}{6} \right)\Bigg|_0^1$$

$= c\dfrac{1}{6}$. Therefore, c = 6.

5-63 a.)

$$f(x) = 6\int_0^{1-x}\int_0^{1-x-y} dzdy = \int_0^{1-x} 6(1-x-y)dy = \left( y - xy - \frac{y^2}{2} \right)\Bigg|_0^{1-x}$$

$$= 6(\frac{x^2}{2} - x + \frac{1}{2}) = 3(x-1)^2 \text{ for } 0 < x < 1$$

b.)

$$f(x,y) = 6\int_0^{1-x-y} dz = 6(1-x-y)$$

$$\text{for } x > 0,\ y > 0 \text{ and } x + y < 1$$

c.)

$$f(x \mid y=0.5, z=0.5) = \frac{f(x, y=0.5, z=0,5)}{f(y=0.5, z=0.5)} = \frac{6}{6} = 1 \text{ For, } x=0$$

d. ) The marginal $f_Y(y)$ is similar to $f_X(x)$ and $f_Y(y) = 3(1-y)^2$ for $0 < y < 1$.

$$f_{X|Y}(x \mid 0.5) = \frac{f(x, 0.5)}{f_Y(0.5)} = \frac{6(0.5-x)}{3(0.25)} = 4(1-2x) \text{ for } x < 0.5$$

5-65. 5-65. a) Let X denote the weight of a brick. Then,

$P(X > 2.75) = P(Z > \frac{2.75-3}{0.25}) = P(Z > -1) = 0.84134$.

Let Y denote the number of bricks in the sample of 20 that exceed 2.75 pounds. Then, by independence, Y has a binomial distribution with n = 20 and p = 0.84134. Therefore, the answer is $P(Y = 20) = \binom{20}{20} 0.84134^{20} = 0.032$.

b) Let A denote the event that the heaviest brick in the sample exceeds 3.75 pounds. Then, P(A) = 1 - P(A') and A' is the event that all bricks weigh less than 3.75 pounds. As in part a., P(X < 3.75) = P(Z < 3) and

$P(A) = 1 - [P(Z < 3)]^{20} = 1 - 0.99865^{20} = 0.0267$.

Section 5-5

5-67. E(X) = 1(3/8)+2(1/2)+4(1/8)=15/8 = 1.875

E(Y) = 3(1/8)+4(1/4)+5(1/2)+6(1/8)=37/8 = 4.625

$$\begin{aligned} \mathrm{E(XY)} &= [1\times 3\times (1/8)] + [1\times 4\times (1/4)] + [2\times 5\times (1/2)] + [4\times 6\times (1/8)] \\ &= 75/8 = 9.375 \end{aligned}$$

$$\sigma_{XY} = E(XY) - E(X)E(Y) = 9.375 - (1.875)(4.625) = 0.703125$$

$V(X) = 1^2(3/8) + 2^2(1/2) + 4^2(1/8) - (15/8)^2 = 0.8594$

$V(Y) = 3^2(1/8) + 4^2(1/4) + 5^2(1/2) + 6^2(1/8) - (15/8)^2 = 0.7344$

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} = \frac{0.703125}{\sqrt{(0.8594)(0.7344)}} = 0.8851$$

5-69.

$$\sum_{x=1}^{3}\sum_{y=1}^{3} c(x+y) = 36c, \qquad c = 1/36$$

$$E(X) = \frac{13}{6} \qquad E(Y) = \frac{13}{6} \qquad E(XY) = \frac{14}{3} \qquad \sigma_{xy} = \frac{14}{3} - \left(\frac{13}{6}\right)^2 = \frac{-1}{36}$$

$$E(X^2) = \frac{16}{3} \qquad E(Y^2) = \frac{16}{3} \qquad V(X) = V(Y) = \frac{23}{36}$$

$$\rho = \frac{\frac{-1}{36}}{\sqrt{\frac{23}{36}}\sqrt{\frac{23}{36}}} = -0.0435$$

5-73. $E(X)=\frac{2}{19}\int_0^1\int_0^{x+1} xdydx+\frac{2}{19}\int_1^5\int_{x-1}^{x+1} xdydx=2.614$

$$E(Y)=\frac{2}{19}\int_0^1\int_0^{x+1} ydydx+\frac{2}{19}\int_1^5\int_{x-1}^{x+1} ydydx=2.649$$

Now, $E(XY)=\frac{2}{19}\int_0^1\int_0^{x+1} xydydx+\frac{2}{19}\int_1^5\int_{x-1}^{x+1} xydydx=8.7763$

$$\sigma_{xy}=8.7763-(2.614)(2.649)=1.85181$$

$$E(X^2)=8.7632 \qquad E(Y^2)=9.07895$$

$$V(x)=1.930, \qquad V(Y)=2.062$$

$$\rho=\frac{1.852}{\sqrt{1.930}\sqrt{2.062}}=0.9279$$

Section 5-6

5-81. Because $\rho = 0$ and X and Y are normally distributed, X and Y are independent. Therefore,

$\mu_X = 0.1$mm $\sigma_X$=0.00031mm $\mu_Y = 0.23$mm $\sigma_Y$=0.00017mm

Probability X is within specification limits is

$$P(0.099535 < X < 0.100465) = P\left(\frac{0.099535-0.1}{0.00031} < Z < \frac{0.100465-0.1}{0.00031}\right)$$
$$= P(-1.5 < Z < 1.5) = P(Z < 1.5) - P(Z < -1.5)$$
$$= 0.8664$$

Probability that Y is within specification limits is

$$P(0.22966 < X < 0.23034) = P\left(\frac{0.22966-0.23}{0.00017} < Z < \frac{0.23034-0.23}{0.00017}\right)$$
$$= P(-2 < Z < 2) = P(Z < 2) - P(Z < -2)$$
$$= 0.9545$$

Probability that a randomly selected lamp is within specification limits is (0.8664)(.9594)=0.8270

Section 5-7

5-87. a) E(2X + 3Y) = 2(0) + 3(10) = 30

b) V(2X + 3Y) = 4V(X) + 9V(Y) = 97

c) 2X + 3Y is normally distributed with mean 30 and variance 97. Therefore,

$$P(2X+3Y<30)=P(Z<\tfrac{30-30}{\sqrt{97}})=P(Z<0)=0.5$$

d) $P(2X+3Y<40)=P(Z<\tfrac{40-30}{\sqrt{97}})=P(Z<1.02)=0.8461$

5-89 a) Let T denote the total thickness. Then, T = X + Y and E(T) = 4 mm,

V(T) = $0.1^2+0.1^2=0.02\text{mm}^2$, and $\sigma_T = 0.1414$ mm.

b)

$$P(T > 4.3) = P\left(Z > \frac{4.3-4}{0.1414}\right) = P(Z > 2.12)$$

2.12) = 1 - 0.983 = 0.017

$$= 1 - P(Z < 2.12) = 1 - 0.983 = 0.0170$$

5-93. a) Let $\overline{X}$ denote the average fill-volume of 100 cans. $\sigma_{\overline{X}} = \sqrt{0.5^2/100} = 0.05$.

b) E($\overline{X}$) = 12.1 and $P(\overline{X} < 12) = P\left(Z < \frac{12-12.1}{0.05}\right) = P(Z < -2) = 0.023$

c) P($\overline{X}$ < 12) = 0.005 implies that $P\left(Z < \frac{12-\mu}{0.05}\right) = 0.005.$

Then $\frac{12-\mu}{0.05}$ = -2.58 and $\mu = 12.129$.

d.) P($\overline{X}$ < 12) = 0.005 implies that $P\left(Z < \frac{12-12.1}{\sigma/\sqrt{100}}\right) = 0.005.$

Then $\frac{12-12.1}{\sigma/\sqrt{100}}$ = -2.58 and $\sigma = 0.388$.

e.) P($\overline{X}$ < 12) = 0.01 implies that $P\left(Z < \frac{12-12.1}{0.5/\sqrt{n}}\right) = 0.01.$

Then $\frac{12-12.1}{0.5/\sqrt{n}}$ = -2.33 and $n = 135.72 \cong 136$.

Supplemental Exercises

5-97. a) $P(X < 0.5, Y < 1.5) = f_{XY}(0,1) + f_{XY}(0,0) = 1/8 + 1/4 = 3/8$.

b) $P(X \le 1) = f_{XY}(0,0) + f_{XY}(0,1) + f_{XY}(1,0) + f_{XY}(1,1) = 3/4$

c) $P(Y < 1.5) = f_{XY}(0,0) + f_{XY}(0,1) + f_{XY}(1,0) + f_{XY}(1,1) = 3/4$

d) $P(X > 0.5, Y < 1.5) = f_{XY}(1,0) + f_{XY}(1,1) = 3/8$

e) E(X) = 0(3/8) + 1(3/8) + 2(1/4) = 7/8.
V(X) = $0^2$(3/8) + $1^2$(3/8) + $2^2$(1/4) - $7/8^2$ =39/64
E(Y) = 1(3/8) + 0(3/8) + 2(1/4) = 7/8.
. V(Y) = $1^2$(3/8) + $0^2$(3/8) + $2^2$(1/4) - $7/8^2$ =39/64

5-105. a) $P(X < 1, Y < 1) = \int_0^1\int_0^1 \frac{1}{18}x^2 y\,dy\,dx = \int_0^1 \frac{1}{18}x^2 \left.\frac{y^2}{2}\right|_0^1 dx = \left.\frac{1}{36}\frac{x^3}{3}\right|_0^1 = \frac{1}{108}$

b) $P(X < 2.5) = \int_0^{2.5}\int_0^2 \frac{1}{18}x^2 y\,dy\,dx = \int_0^{2.5} \frac{1}{18}x^2 \left.\frac{y^2}{2}\right|_0^2 dx = \left.\frac{1}{9}\frac{x^3}{3}\right|_0^{2.5} = 0.5787$

c) $P(1 < Y < 2.5) = \int_0^3\int_1^2 \frac{1}{18}x^2 y\,dy\,dx = \int_0^3 \frac{1}{18}x^2 \left.\frac{y^2}{2}\right|_1^2 dx = \left.\frac{1}{12}\frac{x^3}{3}\right|_0^3 = \frac{3}{4}$

d)

$$P(X>2,1<Y<1.5)=\int_2^3\int_1^{1.5}\tfrac{1}{18}x^2ydydx=\int_2^3\tfrac{1}{18}x^2\left.\tfrac{y^2}{2}\right|_1^{1.5}dx=\left.\tfrac{5}{144}\tfrac{x^3}{3}\right|_2^3$$

$$=\tfrac{95}{432}=0.2199$$

e) $E(X)=\int_0^3\int_0^2\tfrac{1}{18}x^3ydydx=\int_0^3\tfrac{1}{18}x^3 2dx=\left.\tfrac{1}{9}\tfrac{x^4}{4}\right|_0^3=\tfrac{9}{4}$

f) $E(Y)=\int_0^3\int_0^2\tfrac{1}{18}x^2y^2dydx=\int_0^3\tfrac{1}{18}x^2\tfrac{8}{3}dx=\left.\tfrac{4}{27}\tfrac{x^3}{3}\right|_0^3=\tfrac{4}{3}$

5-107. The region $x^2+y^2\le 1$ and $0<z<4$ is a cylinder of radius 1 ( and base area $\pi$) and height 4. Therefore, the volume of the cylinder is $4\pi$ and $f_{XYZ}(x,y,z)=\frac{1}{4\pi}$ for $x^2+y^2\le 1$ and $0<z<4$.

a) The region $X^2+Y^2\le 0.5$ is a cylinder of radius $\sqrt{0.5}$ and height 4. Therefore,

$P(X^2+Y^2\le 0.5)=\tfrac{4(0.5\pi)}{4\pi}=1/2$.

b) The region $X^2+Y^2\le 0.5$ and $0<z<2$ is a cylinder of radius $\sqrt{0.5}$ and height 2. Therefore,

$P(X^2+Y^2\le 0.5, Z<2)=\tfrac{2(0.5\pi)}{4\pi}=1/4$

c) $f_{XY|1}(x,y)=\dfrac{f_{XYZ}(x,y,1)}{f_Z(1)}$ and $f_Z(z)=\iint_{x^2+y^2\le 1}\tfrac{1}{4\pi}dydx=1/4$

for $0<z<4$. Then, $f_{XY|1}(x,y)=\dfrac{1/4\pi}{1/4}=\tfrac{1}{\pi}\ \ for\ x^2+y^2\le 1$.

d) $f_X(x)=\int_0^4\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\tfrac{1}{4\pi}dydz=\int_0^4\tfrac{1}{2\pi}\sqrt{1-x^2}dz=\tfrac{2}{\pi}\sqrt{1-x^2}$ for $-1<x<1$

5-111. Let X, Y, and Z denote the number of problems that result in functional, minor, and no defects, respectively.

a) $P(X=2,Y=5)=P(X=2,Y=5,Z=3)=\tfrac{10!}{2!5!3!}0.2^2 0.5^5 0.3^3=0.085$

b) Z is binomial with n = 10 and p = 0.3.

c) E(Z) = 10(0.3) = 3.

5-115. Let $\overline{X}$ denote the average time to locate 10 parts. Then, E( $\overline{X}$ ) =45 and $\sigma_{\overline{X}}=\frac{30}{\sqrt{10}}$

a) $P(\overline{X}>60)=P(Z>\tfrac{60-45}{30/\sqrt{10}})=P(Z>1.58)=0.057$

b) Let Y denote the total time to locate 10 parts. Then, Y > 600 if and only if $\overline{X}$ > 60. Therefore, the answer is the same as part a.

5-119 Let T denote the total thickness. Then, $T = X_1 + X_2$ and

a.) E(T) = 0.5+1=1.5 mm

$V(T)=V(X_1)+V(X_2)+2Cov(X_1X_2)=0.01+0.04+2(0.14)=0.078mm^2$

where $Cov(XY)=\rho\sigma_X\sigma_Y=0.7(0.1)(0.2)=0.014$

b.) $P(T < 1) = P\left(Z < \frac{1-1.5}{0.078}\right) = P(Z < -6.41) \cong 0$

c.) Let P denote the total thickness. Then, $P = 2X_1 + 3X_2$ and
$E(P) = 2(0.5)+3(1) = 4$ mm
$V(P) = 4V(X_1) + 9V(X_2) + 2(2)(3)Cov(X_1X_2) = 4(0.01)+9(0.04)+2(2)(3)(0.014) = 0.568 mm^2$
where $Cov(XY) = \rho\sigma_X\sigma_Y = 0.7(0.1)(0.2) = 0.014$

5-121 Let X and Y denote the percentage returns for security one and two respectively.
If ½ of the total dollars is invested in each then ½X+ ½Y is the percentage return.
E(½X+ ½Y)=5 million
V(½X+ ½Y)=1/4 V(X)+1/4V(Y)-2(1/2)(1/2)Cov(X,Y)
where $Cov(XY) = \rho\sigma_X\sigma_Y = -0.5(2)(4) = -4$
V(½X+ ½Y)=1/4(4)+1/4(6)-2=3
Also, E(X)=5 and V(X) = 4. Therefore, the strategy that splits between the securities has a lower standard deviation of percentage return.