Chapter 8 Selected Problem Solutions

Section 8-2

8-1 a.) The confidence level for $\bar{x} - 2.14\sigma / \sqrt{n} \le \mu \le \bar{x} + 2.14\sigma / \sqrt{n}$ is determined by the by the value of z_0 which is 2.14. From Table II, we find $\Phi(2.14) = P(Z<2.14) = 0.9793$ and the confidence level is 97.93%.

b.) The confidence level for $\bar{x} - 2.49\sigma / \sqrt{n} \le \mu \le \bar{x} + 2.49\sigma / \sqrt{n}$ is determined by the by the value of z_0 which is 2.14. From Table II, we find $\Phi(2.49) = P(Z<2.49) = 0.9936$ and the confidence level is 99.36%.

c.) The confidence level for $\bar{x} - 1.85\sigma / \sqrt{n} \le \mu \le \bar{x} + 1.85\sigma / \sqrt{n}$ is determined by the by the value of z_0 which is 2.14. From Table II, we find $\Phi(1.85) = P(Z<1.85) = 0.9678$ and the confidence level is 96.78%.

8-7 a.) The 99% CI on the mean calcium concentration would be longer.

b). No, that is not the correct interpretation of a confidence interval. The probability that μ is between 0.49 and 0.82 is either 0 or 1.

c). Yes, this is the correct interpretation of a confidence interval. The upper and lower limits of the confidence limits are random variables.

8-13 a) 95% two sided CI on the mean compressive strength

$z_{\alpha/2} = z_{0.025} = 1.96$, and $\bar{x} = 3250$, $\sigma^2 = 1000$, $n=12$

$$\bar{x} - z_{0.025}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.025}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$3250 - 1.96\left(\frac{31.62}{\sqrt{12}}\right) \le \mu \le 3250 + 1.96\left(\frac{31.62}{\sqrt{12}}\right)$$

$$3232.11 \le \mu \le 3267.89$$

b.) 99% Two-sided CI on the true mean compressive strength

$z_{\alpha/2} = z_{0.005} = 2.58$

$$\bar{x} - z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$3250 - 2.58\left(\frac{31.62}{\sqrt{12}}\right) \le \mu \le 3250 + 2.58\left(\frac{31.62}{\sqrt{12}}\right)$$

$$3226.5 \le \mu \le 3273.5$$

8-15 Set the width to 6 hours with $\sigma = 25$, $z_{0.025} = 1.96$ solve for n.

$1/2 \text{ width} = (1.96)(25) / \sqrt{n} = 3$

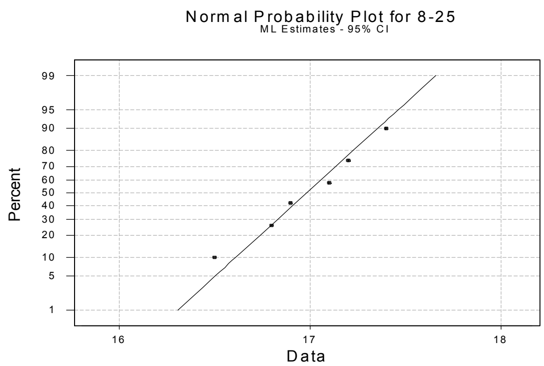
$49 = 3\sqrt{n}$

$n = \left(\frac{49}{3}\right)^2 = 266.78$

Therefore, n=267.

Section 8-3

8-25 a.) The data appear to be normally distributed based on examination of the normal probability plot below. Therefore, there is evidence to support that the level of polyunsaturated fatty acid is normally distributed.



b.) 99% CI on the mean level of polyunsaturated fatty acid.
For $\alpha = 0.01$, $t_{\alpha/2,n-1} = t_{0.005,5} = 4.032$

$$\bar{x} - t_{0.005,5}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.005,5}\left(\frac{s}{\sqrt{n}}\right)$$

$$16.98 - 4.032\left(\frac{0.319}{\sqrt{6}}\right) \le \mu \le 16.98 + 4.032\left(\frac{0.319}{\sqrt{6}}\right)$$

$$16.455 \le \mu \le 17.505$$

8-29 95% lower bound confidence for the mean wall thickness
given $\bar{x} = 4.05$ $s = 0.08$ $n = 25$

$t_{\alpha,n-1} = t_{0.05,24} = 1.711$

$$\bar{x} - t_{0.05,24}\left(\frac{s}{\sqrt{n}}\right) \le \mu$$

$$4.05 - 1.711\left(\frac{0.08}{\sqrt{25}}\right) \le \mu$$

$$4.023 \le \mu$$

It may be assumed that the mean wall thickness will most likely be greater than 4.023 mm.

8-31 $\bar{x} = 1.10$ $s = 0.015$ $n = 25$

95% CI on the mean volume of syrup dispensed

For $\alpha = 0.05$ and n = 25, $t_{\alpha/2,n-1} = t_{0.025,24} = 2.064$

$$\bar{x} - t_{0.025,24}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,24}\left(\frac{s}{\sqrt{n}}\right)$$

$$1.10 - 2.064\left(\frac{0.015}{\sqrt{25}}\right) \le \mu \le 1.10 + 2.064\left(\frac{0.015}{\sqrt{25}}\right)$$

$$1.093 \le \mu \le 1.106$$

Section 8-4

8-35 99% lower confidence bound for σ^2

For $\alpha = 0.01$ and n = 15, $\chi^2_{\alpha,n-1} = \chi^2_{0.01,14} = 29.14$

$$\frac{14(0.008)^2}{29.14} < \sigma^2$$

$$0.00003075 < \sigma^2$$

8-37 95% lower confidence bound for σ^2 given n = 16, $s^2 = (3645.94)^2$

For $\alpha = 0.05$ and n = 16, $\chi^2_{\alpha,n-1} = \chi^2_{0.05,15} = 25$

$$\frac{15(3645.94)^2}{25} < \sigma^2$$

$$7{,}975{,}727.09 < \sigma^2$$

8-39 95% confidence interval for σ: given n = 51, s = 0.37

First find the confidence interval for σ^2 :

For $\alpha = 0.05$ and n = 51, $\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,50} = 71.42$ and $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,50} = 32.36$

$$\frac{50(0.37)^2}{(71.42)^2} \le \sigma^2 \le \frac{50(0.37)^2}{(32.36)^2}$$

$$0.096 \le \sigma^2 \le 0.2115$$

Taking the square root of the endpoints of this interval we obtain,

$$0.31 < \sigma < 0.46$$

8-41 90% lower confidence bound on σ (the standard deviation of the sugar content) given n = 10, $s^2 = 23.04$

For $\alpha = 0.1$ and n = 10, $\chi^2_{\alpha,n-1} = \chi^2_{0.1,9} = 19.02$

$$\frac{9(23.04)}{14.68} \le \sigma^2$$

$$14.13 \le \sigma^2$$

Take the square root of the endpoints of this interval to find the confidence interval for σ:

$$3.8 \le \sigma$$

Section 8-7

8-63 99% tolerance interval on the polyunsaturated fatty acid in this type of margarine that has a confidence level of 95%

$\bar{x} = 16.98$ s = 0.319 n=6 and k = 5.775

$$\bar{x} - ks, \bar{x} + ks$$

$$16.98 - 5.775(0.319), 16.98 + 5.775(0.319)$$

$$(15.14, 18.82)$$

The 99% tolerance interval is much wider than the 99% confidence interval on the population mean $(16.46 \le \mu \le 17.51)$.

8-67 90% lower tolerance bound on bottle wall thickness that has confidence level 90%.

given $\bar{x} = 4.05$ s = 0.08 n = 25 and k = 1.702

$$\bar{x} - ks$$

$$4.05 - 1.702(0.08)$$

$$3.91$$

$$\text{The 90\% tolarance bound is } (3.91, \infty)$$

The lower tolerance bound is of interest if we want to make sure the wall thickness is at least a certain value so that the bottle will not break.

8-69 95% tolerance interval on the syrup volume that has 90% confidence level

$\bar{x} = 1.10$ s = 0.015 n = 25 and k=2.474

$$\bar{x} - ks, \bar{x} + ks$$

$$1.10 - 2.474(0.015), 1.10 + 2.474(0.015)$$

$$(1.06, 1.14)$$

Supplemental Exercises

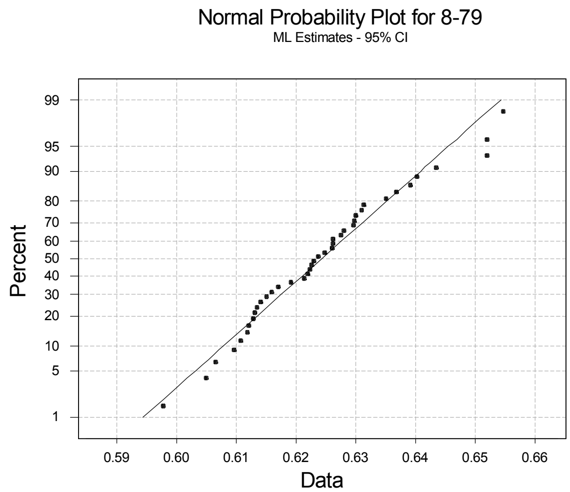
8-75 With $\sigma = 8$, the 95% confidence interval on the mean has length of at most 5; the error is then E = 2.5.

a) $n = \left(\frac{z_{0.025}}{2.5}\right)^2 8^2 = \left(\frac{1.96}{2.5}\right)^2 64 = 39.34 = 40$

b) $n = \left(\frac{z_{0.025}}{2.5}\right)^2 6^2 = \left(\frac{1.96}{2.5}\right)^2 36 = 22.13 = 23$

As the standard deviation decreases, with all other values held constant, the sample size necessary to maintain the acceptable level of confidence and the length of the interval, decreases.

8-79 Normal probability plot for the coefficient of restitution



b.) 99% CI on the true mean coefficient of restitution

$\bar{x} = 0.624$, $s = 0.013$, $n = 40$ $t_{a/2,\,n-1} = t_{0.005,\,39} = 2.7079$

$$\bar{x} - t_{0.005,39}\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + t_{0.005,39}\frac{s}{\sqrt{n}}$$

$$0.624 - 2.7079\frac{0.013}{\sqrt{40}} \le \mu \le 0.624 + 2.7079\frac{0.013}{\sqrt{40}}$$

$$0.618 \le \mu \le 0.630$$

b.) 99% prediction interval on the coefficient of restitution for the next baseball that will be tested.

$$\bar{x} - t_{0.005,39}s\sqrt{1+\frac{1}{n}} \le x_{n+1} \le \bar{x} + t_{0.005,39}s\sqrt{1+\frac{1}{n}}$$

$$0.624 - 2.7079(0.013)\sqrt{1+\frac{1}{40}} \le x_{n+1} \le 0.624 + 2.7079(0.013)\sqrt{1+\frac{1}{40}}$$

$$0.588 \le x_{n+1} \le 0.660$$

c.) 99% tolerance interval on the coefficient of restitution with a 95% level of confidence

$$(\bar{x} - ks,\ \bar{x} + ks)$$

$$(0.624 - 3.213(0.013),\ 0.624 + 3.213(0.013))$$

$$(0.583,\ 0.665)$$

e.)The confidence interval in part (b) describes the confidence interval on the population mean and we may interpret this to mean that 99% of such intervals will cover the population mean. The prediction interval tells us that within that within a 99% probability that the next baseball will have a coefficient of restitution between 0.588 and 0.660. The tolerance interval captures 99% of the values of the normal distribution with a 95% level of confidence.

8-83 a.) 95% Confidence Interval on the population proportion

n=1200 x=8 $\hat{p} = 0.0067$ $z_{\alpha/2}=z_{0.025}=1.96$

$$\hat{p} - z_{a/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{a/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.0067 - 1.96\sqrt{\frac{0.0067(1-0.0067)}{1200}} \le p \le 0.0067 + 1.96\sqrt{\frac{0.0067(1-0.0067)}{1200}}$$

$$0.0021 \le p \le 0.0088$$

b.) Yes, there is evidence to support the claim that the fraction of defective units produced is one percent or less. This is true because the confidence interval does not include 0.01 and the upper limit of the control interval is lower than 0.01.