Chapter 9 Selected Problems Solutions

Section 9-1

9-1 a) $H_0 : \mu = 25, H_1 : \mu \neq 25$ Yes, because the hypothesis is stated in terms of the parameter of interest, inequality is in the alternative hypothesis, and the value in the null and alternative hypotheses matches.

b) $H_0 : \sigma > 10, H_1 : \sigma = 10$ No, because the inequality is in the null hypothesis.

c) $H_0 : \bar{x} = 50, H_1 : \bar{x} \neq 50$ No, because the hypothesis is stated in terms of the statistic rather than the parameter.

d) $H_0 : p = 0.1, H_1 : p = 0.3$ No, the values in the hull and alternative hypotheses do not match and both of the hypotheses are equality statements.

e) $H_0 : s = 30, H_1 : s > 30$ No, because the hypothesis is stated in terms of the statistic rather than the parameter.

9-3 a) $\alpha = P(\bar{X} \leq 11.5 \mid \mu = 12) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \leq \frac{11.5-12}{0.5/\sqrt{16}}\right) = P(Z \leq -4) = 1 - P(Z \leq 4)$

$= 1 - 1 = 0.$

The probability of rejecting the null, when the null is true, is approximately 0 with a sample size of 16.

b) $\beta = P(\bar{X} > 11.5 \mid \mu = 11.25) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{11.5-11.25}{0.5/\sqrt{16}}\right) = P(Z > 2) = 1 - P(Z \leq 2)$

$= 1 - 0.97725 = 0.02275.$

The probability of accepting the null hypothesis when it is false is 0.02275.

9-9 a) $z = \frac{190-175}{20/\sqrt{10}} = 2.37$, Note that z is large, therefore **reject** the null hypothesis and conclude that the mean foam height is greater than 175 mm.

b) $P(\bar{X} > 190 \text{ when } \mu = 175)$

$= P\left(\frac{\bar{X}-175}{20/\sqrt{10}} > \frac{190-175}{20/\sqrt{10}}\right)$

$= P(Z > 2.37) = 1 - P(Z \leq 2.37)$

$= 1 - 0.99111$

$= 0.00889.$

The probability that a value of at least 190 mm would be observed (if the true mean height is 175 mm) is only 0.00889. Thus, the sample value of $\bar{x} = 190$ mm would be an unusual result.

9-17. The problem statement implies H_0: $p = 0.6$, H_1: $p > 0.6$ and defines an acceptance region as $\hat{p} \leq \frac{315}{500} = 0.63$ and rejection region as $\hat{p} > 0.63$

a) $\alpha = P(\hat{P} \geq 0.63 \mid p = 0.6) = P\left(Z \geq \frac{0.63 - 0.6}{\sqrt{\frac{0.6(0.4)}{500}}}\right).$

$= P(Z \geq 1.37) = 1 - P(Z < 1.37) = 0.08535$

b) $\beta = P(\hat{P} \leq 0.63 \text{ when } p = 0.75) = P(Z \leq -6.196) \cong 0.$

Section 9-2

9-21. a) 1) The parameter of interest is the true mean yield, μ.

2) $H_0 : \mu = 90$

3) $H_1 : \mu \neq 90$

4) $\alpha = 0.05$

5) $z_0 = \dfrac{\overline{x} - \mu}{\sigma / \sqrt{n}}$

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{0.025} = -1.96$ or $z_0 > z_{\alpha/2}$ where $z_{0.025} = 1.96$

7) $\overline{x} = 90.48$, $\sigma = 3$

$$z_0 = \frac{90.48 - 90}{3/\sqrt{5}} = 0.36$$

8) Since $-1.96 < 0.36 < 1.96$ do not reject H_0 and conclude the yield is not significantly different from 90% at $\alpha = 0.05$.

b) P-value $= 2[1 - \Phi(0.36)] = 2[1 - 0.64058] = 0.71884$

c) $n = \dfrac{(z_{\alpha/2} + z_\beta)^2 \sigma^2}{\delta^2} = \dfrac{(z_{0.025} + z_{0.05})^2 3^2}{(85 - 90)^2} = \dfrac{(1.96 + 1.65)^2 9}{(-5)^2} = 4.67$

$n \cong 5$.

d) $\beta = \Phi\left(z_{0.025} + \dfrac{90 - 92}{3/\sqrt{5}}\right) - \Phi\left(-z_{0.025} + \dfrac{90 - 92}{3/\sqrt{5}}\right)$

$= \Phi(1.96 + -1.491) - \Phi(-1.96 + -1.491)$

$= \Phi(0.47) - \Phi(-3.45)$

$= \Phi(0.47) - (1 - \Phi(3.45))$

$= 0.68082 - (1 - 0.99972)$

$= 0.68054$.

e) For $\alpha = 0.05$, $z_{\alpha/2} = z_{0.025} = 1.96$

$\overline{x} - z_{0.025}\left(\dfrac{\sigma}{\sqrt{n}}\right) \le \mu \le \overline{x} + z_{0.025}\left(\dfrac{\sigma}{\sqrt{n}}\right)$

$90.48 - 1.96\left(\dfrac{3}{\sqrt{5}}\right) \le \mu \le 90.48 + 1.96\left(\dfrac{3}{\sqrt{5}}\right)$

$87.85 \le \mu \le 93.11$

With 95% confidence, we believe the true mean yield of the chemical process is between 87.85% and 93.11%.

9-25. a) 1) The parameter of interest is the true mean tensile strength, μ.

2) $H_0 : \mu = 3500$

3) $H_1 : \mu \neq 3500$

4) $\alpha = 0.01$

5) $z_0 = \dfrac{\overline{x} - \mu}{\sigma / \sqrt{n}}$

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{0.005} = -2.58$ or $z_0 > z_{\alpha/2}$ where $z_{0.005} = 2.58$

7) $\overline{x} = 3250$, $\sigma = 60$

$$z_0 = \frac{3250 - 3500}{60/\sqrt{12}} = -14.43$$

8) Since $-14.43 < -2.58$, reject the null hypothesis and conclude the true mean compressive strength is significantly different from 3500 at $\alpha = 0.01$.

b) Smallest level of significance = P-value $= 2[1 - \Phi(14.43)] = 2[1 - 1] = 0$

The smallest level of significance at which we are willing to reject the null hypothesis is 0.

c) $z_{\alpha/2} = z_{0.025} = 1.96$

$$\bar{x} - z_{0.025}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.025}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$3250 - 1.96\left(\frac{31.62}{\sqrt{12}}\right) \le \mu \le 3250 + 1.96\left(\frac{31.62}{\sqrt{12}}\right)$$

$$3232.11 \le \mu \le 3267.89$$

With 95% confidence, we believe the true mean tensile strength is between 3232.11 psi and 3267.89 psi. We can test the hypotheses that the true mean strength is not equal to 3500 by noting that the value is not within the confidence interval.

9-27 a) 1) The parameter of interest is the true mean speed, μ.

2) $H_0 : \mu = 100$

3) $H_1 : \mu < 100$

4) $\alpha = 0.05$

5) $z_0 = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

6) Reject H_0 if $z_0 < -z_\alpha$ where $-z_{0.05} = -1.65$

7) $\bar{x} = 102.2$, $\sigma = 4$

$$z_0 = \frac{102.2 - 100}{4/\sqrt{8}} = 1.55$$

8) Since $1.55 > -1.65$, do not reject the null hypothesis and conclude the there is insufficient evidence to conclude that the true speed strength is less than 100 at $\alpha = 0.05$.

b) $\beta = \Phi\left(-z_{0.05} - \dfrac{(95-100)\sqrt{8}}{4}\right) = \Phi(-1.65 - -3.54) = \Phi(1.89) = 1$

Power = $1-\beta = 1-0.97062 = 0.02938$

c) $n = \dfrac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta^2} = \dfrac{(z_{0.05} + z_{0.15})^2 \sigma^2}{(95-100)^2} = \dfrac{(1.65+1.03)^2 (4)^2}{(5)^2} = 0.927,$

$n \cong 1$

d) $\bar{x} - z_{0.05}\left(\dfrac{\sigma}{\sqrt{n}}\right) \le \mu$

$$102.2 - 1.65\left(\frac{4}{\sqrt{8}}\right) \le \mu$$

$$99.866 \le \mu$$

Since the lower limit of the CI is just slightly below 100, we are confident that the mean speed is not less than 100 m/s.

9-29 a) 1) The parameter of interest is the true average battery life, μ.

2) $H_0 : \mu = 4$

3) $H_1 : \mu > 4$

4) $\alpha = 0.05$

5) $z_0 = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

6) Reject H_0 if $z_0 > z_\alpha$ where $z_{0.05} = 1.65$

7) $\bar{x} = 4.05$, $\sigma = 0.2$

$$z_0 = \frac{4.05 - 4}{0.2/\sqrt{50}} = 1.77$$

8) Since 1.77>1.65, reject the null hypothesis and conclude that the there is sufficient evidence to conclude that the true average battery life exceeds 4 hours at $\alpha = 0.05$.

b) $\beta = \Phi\left(z_{0.05} - \frac{(4.5-4)\sqrt{50}}{0.2} \right)$ = Φ(1.65 – 17.68) = Φ(-16.03) = 0

Power = 1-β = 1-0 = 1

c) n = $\frac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta^2} = \frac{(z_{0.05} + z_{0.1})^2 \sigma^2}{(4.5-4)^2} = \frac{(1.65+1.29)^2 (0.2)^2}{(0.5)^2} = 34.7,$

n ≅ 35

d) $\bar{x} - z_{0.05}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu$

$4.05 - 1.65\left(\frac{0.2}{\sqrt{50}}\right) \le \mu$

$4.003 \le \mu$

Since the lower limit of the CI is just slightly above 4, we conclude that average life is greater than 4 hours at α=0.05.

Section 9-3

9-31 a. 1) The parameter of interest is the true mean female body temperature, μ.

2) $H_0 : \mu = 98.6$

3) $H_1 : \mu \ne 98.6$

4) $\alpha = 0.05$

5) $t_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}}$

6) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where $t_{\alpha/2,n-1} = 2.064$

7) $\bar{x} = 98.264$, s = 0.4821 n=25

$$t_0 = \frac{98.264 - 98.6}{0.4821/\sqrt{25}} = -3.48$$

8) Since 3.48 > 2.064, reject the null hypothesis and conclude that the there is sufficient evidence to conclude that the true mean female body temperature is not equal to 98.6 °F at $\alpha = 0.05$.

P-value = 2* 0.001 = 0.002

b) $d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|98 - 98.6|}{0.4821} = 1.24$

Using the OC curve, Chart VI e) for $\alpha = 0.05$, d = 1.24, and n = 25, we get $\beta \cong 0$ and power of 1–0 ≅ 1.

c) $d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|98.2 - 98.6|}{0.4821} = 0.83$

Using the OC curve, Chart VI g) for $\alpha = 0.05$, d = 0.83, and $\beta \cong 0.1$ (Power=0.9),

$n^* = 20$. Therefore, $n = \frac{n^*+1}{2} = \frac{20+1}{2} = 10.5$ and n=11.

d) 95% two sided confidence interval

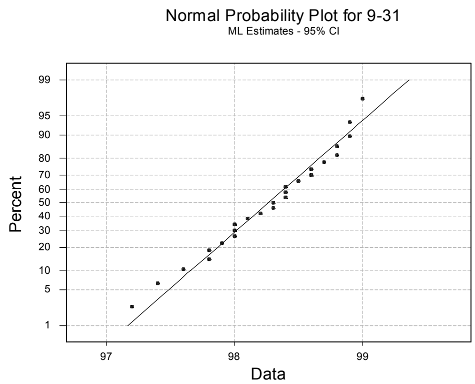
$$\bar{x} - t_{0.025,24}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,24}\left(\frac{s}{\sqrt{n}}\right)$$

$$98.264 - 2.064\left(\frac{0.4821}{\sqrt{25}}\right) \le \mu \le 98.264 + 2.064\left(\frac{0.4821}{\sqrt{25}}\right)$$

$$98.065 \le \mu \le 98.463$$

We can conclude that the mean female body temperature is not equal to 98.6 since the value is not included inside the confidence interval.

e)



Data appear to be normally distributed.

9-37. a.) In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true mean coefficient of restitution, μ.

2) $H_0 : \mu = 0.635$

3) $H_1 : \mu > 0.635$

4) $\alpha = 0.05$

5) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,39} = 1.685$

7) $\bar{x} = 0.624$ $s = 0.013$ $n = 40$

$$t_0 = \frac{0.624 - 0.635}{0.013/\sqrt{40}} = -5.35$$

8) Since –5.25 < 1.685, do not reject the null hypothesis and conclude that there is not sufficient evidence to indicate that the true mean coefficient of restitution is greater than 0.635 at $\alpha = 0.05$.

b.)The P-value > 0.4, based on Table IV. Minitab gives *P-value* = 1.

c) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|0.64 - 0.635|}{0.013} = 0.38$

Using the OC curve, Chart VI g) for $\alpha = 0.05$, d = 0.38, and n = 40, we get $\beta \cong 0.25$ and power of 1–0.25 = 0.75.

d) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|0.638 - 0.635|}{0.013} = 0.23$

Using the OC curve, Chart VI g) for $\alpha = 0.05$, d = 0.23, and $\beta \cong 0.25$ (Power=0.75),

$n^* = 75$. Therefore, $n = \dfrac{n^*+1}{2} = \dfrac{75+1}{2} = 38$ and n=38.

9-41 a) In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true mean concentration of suspended solids, μ.
2) $H_0 : \mu = 55$
3) $H_1 : \mu \neq 55$
4) $\alpha = 0.05$
5) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$
6) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where $t_{0.025,59} = 2.000$
7) $\bar{x} = 59.87$ s = 12.50 n = 60

$$t_0 = \frac{59.87 - 55}{12.50/\sqrt{60}} = 3.018$$

8) Since 3.018 > 2.000, reject the null hypothesis and conclude that there is sufficient evidence to indicate that the true mean concentration of suspended solids is not equal to 55 at $\alpha = 0.05$.

b) From table IV the t0 value is found between the values of 0.001 and 0.0025 with 59 degrees of freedom, so 2*0.001<*P-value* = 2* 0.0025 Therefore, 0.002< P-value<0.005.
Minitab gives a p-value of 0.0038

c) $d = \dfrac{|50 - 55|}{12.50} = 0.4$, n=60 so, from the OC Chart VI e) for $\alpha = 0.05$, d= 0.4 and n=60 we find that $\beta \cong 0.2$. Therefore, the power = 1-0.2 = 0.8.

d) From the same OC chart, and for the specified power, we would need approximately 38 observations.

$d = \dfrac{|50 - 55|}{12.50} = 0.4$ Using the OC Chart VI e) for $\alpha = 0.05$, d = 0.4, and $\beta \cong 0.10$ (Power=0.90),

$n^* = 75$. Therefore, $n = \dfrac{n^*+1}{2} = \dfrac{75+1}{2} = 38$ and n=38.

Section 9-4

9-43 a) In order to use the χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true standard deviation of the diameter, σ. However, the answer can be found by performing a hypothesis test on σ^2.
2) $H_0 : \sigma^2 = 0.0001$
3) $H_1 : \sigma^2 > 0.0001$
4) $\alpha = 0.01$
5) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$
6) Reject H_0 if $\chi_0^2 > \chi_{\alpha,n-1}^2$ where $\chi_{0.01,14}^2 = 29.14$
7) n = 15, $s^2 = 0.008$

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{14(0.008)^2}{0.0001} = 8.96$$

8) Since 8.96 < 29.14 do not reject H_0 and conclude there is insufficient evidence to indicate the true standard deviation of the diameter exceeds 0.01 at $\alpha = 0.01$.

b) P-value = $P(\chi^2 > 8.96)$ for 14 degrees of freedom: 0.5 < P-value < 0.9

c) $\lambda = \dfrac{\sigma}{\sigma_0} = \dfrac{0.0125}{0.01} = 1.25$ power = 0.8, β=0.2

using chart VIk, the required sample size is 50

9-47. a) In order to use χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true standard deviation of titanium percentage, σ. However, the answer can be found by performing a hypothesis test on σ^2.

2) H_0 : $\sigma^2 = (0.25)^2$

3) H_1 : $\sigma^2 \neq (0.25)^2$

4) $\alpha = 0.01$

5) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

6) Reject H_0 if $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$ where $\chi^2_{0.995,50} = 27.99$ or $\chi_0^2 > \chi^2_{\alpha,2,n-1}$ where $\chi^2_{0.005,50} = 79.49$

7) n = 51, s = 0.37

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{50(0.37)^2}{(0.25)^2} = 109.52$$

8) Since 109.52 > 79.49 we would reject H_0 and conclude there is sufficient evidence to indicate the true standard deviation of titanium percentage is significantly different from 0.25 at $\alpha = 0.01$.

b) 95% confidence interval for σ:

First find the confidence interval for σ^2 :

For $\alpha = 0.05$ and n = 51, $\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,50} = 71.42$ and $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,50} = 32.36$

$$\frac{50(0.37)^2}{(71.42)^2} \le \sigma^2 \le \frac{50(0.37)^2}{(32.36)^2}$$

$$0.096 \le \sigma^2 \le 0.2115$$

Taking the square root of the endpoints of this interval we obtain,

$$0.31 < \sigma < 0.46$$

Since 0.25 falls below the lower confidence bound we would conclude that the population standard deviation is not equal to 0.25.

9-49 Using the chart in the Appendix, with $\lambda = \sqrt{\dfrac{40}{18}} = 1.49$ and $\beta = 0.10$, we find n = 30.

Section 9-5

9-51 p= 0.15, p_0=0.10, n=85, and $z_{\alpha/2}$=1.96

$$\beta = \Phi\left(\frac{p_0 - p + z_{\alpha/2}\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right) - \Phi\left(\frac{p_0 - p - z_{\alpha/2}\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right)$$

$$= \Phi\left(\frac{0.10 - 0.15 + 1.96\sqrt{0.10(1-0.10)/85}}{\sqrt{0.15(1-0.15)/85}}\right) - \Phi\left(\frac{0.10 - 0.15 - 1.96\sqrt{0.10(1-0.10)/85}}{\sqrt{0.15(1-0.15)/85}}\right)$$

$$= \Phi(0.36) - \Phi(-2.94) = 0.6406 - 0.0016 = 0.639$$

$$n=\left(\frac{z_{\alpha/2}\sqrt{p_0(1-p_0)}-z_{\beta}\sqrt{p(1-p)}}{p-p_0}\right)^2$$
$$=\left(\frac{1.96\sqrt{0.10(1-0.10)}-1.28\sqrt{0.15(1-0.15)}}{0.15-0.10}\right)^2$$
$$=(10.85)^2=117.63\cong 118$$

9-53. a) Using the information from Exercise 8-51, test

2) $H_0 : p = 0.05$

3) $H_1 : p < 0.05$

4) $\alpha = 0.05$

5) $z_0=\frac{x-np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0=\frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion

6) Reject H_0 if $z_0 < -z_\alpha$ where $-z_\alpha = -z_{0.05} = -1.65$

7) x = 13 n = 300 $\hat{p} = \frac{13}{300} = 0.043$

$$z_0=\frac{x-np_0}{\sqrt{np_0(1-p_0)}}=\frac{13-300(0.05)}{\sqrt{300(0.05)(0.95)}}=-0.53$$

8) Since $-0.53 > -1.65$, do not null hypothesis and conclude the true fraction of defective integrated circuits is not significantly less than 0.05, at $\alpha = 0.05$.

b) P-value = $1 - \Phi(0.53) = 0.29806$

9-57. The problem statement implies that H_0: p = 0.6, H_1: p > 0.6 and defines an acceptance region as $\hat{p} \le \frac{315}{500} = 0.63$ and rejection region as $\hat{p} > 0.63$

a) The probability of a type 1 error is

$$\alpha = P(\hat{p}\ge 0.63 \mid p=0.6)=P\left(Z\ge\frac{0.63-0.6}{\sqrt{\frac{0.6(0.4)}{500}}}\right)=P(Z\ge 1.37)=1-P(Z<1.37)=0.08535$$

b) $\beta = P(\hat{P} \le 0.63 \mid p = 0.75) = P(Z \le -6.196) = 0.$

Section 9-7

9-59.

Value	0	1	2	3	4
Observed Frequency	24	30	31	11	4
Expected Frequency	30.12	36.14	21.69	8.67	2.60

Since value 4 has an expected frequency less than 3, combine this category with the previous category:

Value	0	1	2	3-4
Observed Frequency	24	30	31	15
Expected Frequency	30.12	36.14	21.69	11.67

The degrees of freedom are $k - p - 1 = 4 - 0 - 1 = 3$

a) 1) The variable of interest is the form of the distribution for X.
2) H_0: The form of the distribution is Poisson
3) H_1: The form of the distribution is not Poisson
4) $\alpha = 0.05$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_o^2 > \chi_{0.05,3}^2 = 7.81$

7) $$\chi_0^2 = \frac{(24-30.12)^2}{30.12} + \frac{(30-36.14)^2}{36.14} + \frac{(31-21.69)^2}{21.69} + \frac{(15-11.67)^2}{11.67} = 7.23$$

8) Since 7.23 < 7.81 do not reject H_0. We are unable to reject the null hypothesis that the distribution of X is Poisson.

b) The P-value is between 0.05 and 0.1 using Table III. P-value = 0.0649 (found using Minitab)

9-63 The value of p must be estimated. Let the estimate be denoted by $\hat{p}_{sample}$

$$\text{sample mean} = \frac{0(39) + 1(23) + 2(12) + 3(1)}{75} = 0.6667$$

$$\hat{p}_{sample} = \frac{sample \quad mean}{n} = \frac{0.6667}{24} = 0.02778$$

Value	0	1	2	3
Observed	39	23	12	1
Expected	38.1426	26.1571	8.5952	1.8010

Since value 3 has an expected frequency less than 3, combine this category with that of value 2:

Value	0	1	2-3
Observed	39	23	13
Expected	38.1426	26.1571	10.3962

The degrees of freedom are $k - p - 1 = 3 - 1 - 1 = 1$

a) 1) The variable of interest is the form of the distribution for the number of under-filled cartons, X.
2) H_0: The form of the distribution is binomial
3) H_1: The form of the distribution is not binomial
4) $\alpha = 0.05$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_o^2 > \chi_{0.05,1}^2 = 3.84$

7) $$\chi_0^2 = \frac{(39 - 38.1426)^2}{38.1426} + \frac{(23 - 26.1571)^2}{26.1571} + \frac{(13 - 10.3962)^2}{10.39} = 1.053$$

8) Since 1.053 < 3.84 do not reject H_0. We are unable to reject the null hypothesis that the distribution of the number of under-filled cartons is binomial at $\alpha = 0.05$.

b) The P-value is between 0.5 and 0.1 using Table III P-value = 0.3048 (found using Minitab)

Section 9-8

9-65. 1. The variable of interest is breakdowns among shift.
2. H_0: Breakdowns are independent of shift.

3. H_1: Breakdowns are not independent of shift.
4. $\alpha = 0.05$
5. The test statistic is:

$$\chi_0^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{(O_{ij}-E_{ij})^2}{E_{ij}}$$

6. The critical value is $\chi^2_{.05,6} = 12.592$
7. The calculated test statistic is $\chi^2_0 = 11.65$
8. $\chi_0^2 \not> \chi^2_{0.05,6}$, do not reject H_0 and conclude that the data provide insufficient evidence to claim that machine breakdown and shift are dependent at $\alpha = 0.05$.
P-value = 0.070 (using Minitab)

9-69. 1. The variable of interest is failures of an electronic component.
2. H_0: Type of failure is independent of mounting position.
3. H_1: Type of failure is not independent of mounting position.
4. $\alpha = 0.01$
5. The test statistic is:

$$\chi_0^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{(O_{ij}-E_{ij})^2}{E_{ij}}$$

6. The critical value is $\chi^2_{.01,3} = 11.344$
7. The calculated test statistic is $\chi^2_0 = 10.71$
8. $\chi_0^2 \not> \chi^2_{0.01,3}$, do not reject H_0 and conclude that the evidence is not sufficient to claim that the type of failure is not independent of the mounting position at $\alpha = 0.01$.
P-value = 0.013

Supplemental

9-75. $\sigma = 8$, $\delta = 204 - 200 = -4$, $\frac{\alpha}{2} = 0.025$, $z_{0.025} = 1.96$.

a) n = 20: $\beta = \Phi\left(1.96 - \frac{4\sqrt{20}}{8}\right) = \Phi(-0.28) = 1 - \Phi(0.28) = 1 - 0.61026 = 0.38974$

Therefore, power = $1 - \beta = 0.61026$

b) n = 50: $\beta = \Phi\left(1.96 - \frac{4\sqrt{50}}{8}\right) = \Phi(-2.58) = 1 - \Phi(2.58) = 1 - 0.99506 = 0.00494$

Therefore, power = $1 - \beta = 0.995$

c) n = 100: $\beta = \Phi\left(1.96 - \frac{4\sqrt{100}}{8}\right) = \Phi(-3.04) = 1 - \Phi(3.04) = 1 - 0.99882 = 0.00118$

Therefore, power = $1 - \beta = 0.9988$

d) As sample size increases, and all other values are held constant, the power increases because the variance of the sample mean decreases. Consequently, the probability of a Type II error decreases, which implies the power increases.

9-77. a) Rejecting a null hypothesis provides a *stronger conclusion* than failing to reject a null hypothesis. Therefore, place what we are trying to demonstrate in the alternative hypothesis.

Assume that the data follow a normal distribution.

b) 1) the parameter of interest is the mean weld strength, μ.
2) $H_0 : \mu = 150$
3) $H_1 : \mu > 150$
4) Not given
5) The test statistic is:

$$t_0 = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

6) Since no critical value is given, we will calculate the P-value
7) $\bar{x} = 153.7$, s= 11.3, n=20

$$t_0 = \frac{153.7 - 150}{11.3\sqrt{20}} = 1.46$$

P-value = $P(t \geq 1.46) = 0.05 < p - \text{value} < 0.10$

8) There is some modest evidence to support the claim that the weld strength exceeds 150 psi. If we used $\alpha = 0.01$ or 0.05, we would not reject the null hypothesis, thus the claim would not be supported. If we used $\alpha = 0.10$, we would reject the null in favor of the alternative and conclude the weld strength exceeds 150 psi.

9-79 a) 1) the parameter of interest is the standard deviation, σ
2) $H_0 : \sigma^2 = 400$
3) $H_1 : \sigma^2 < 400$
4) Not given
5) The test statistic is: $\chi_0^2 = \frac{(n-1)s^2}{\sigma^2}$
6) Since no critical value is given, we will calculate the p-value
7) n = 10, s = 15.7

$$\chi_0^2 = \frac{9(15.7)^2}{400} = 5.546$$

P-value = $P(\chi^2 < 5.546)$; $0.1 < P - \text{value} < 0.5$

8) The P-value is greater than any acceptable significance level, α, therefore we do not reject the null hypothesis. There is insufficient evidence to support the claim that the standard deviation is less than 20 microamps.

b) 7) n = 51, s = 20

$$\chi_0^2 = \frac{50(15.7)^2}{400} = 30.81$$

P-value = $P(\chi^2 < 30.81)$; $0.01 < P - \text{value} < 0.025$

8) The P-value is less than 0.05, therefore we reject the null hypothesis and conclude that the standard deviation is significantly less than 20 microamps.

c) Increasing the sample size increases the test statistic χ_0^2 and therefore decreases the P-value, providing more evidence against the null hypothesis.

9-85 We can divide the real line under a standard normal distribution into eight intervals with equal probability. These intervals are [0,.32), [0.32, 0.675), [0.675, 1.15), [1.15, ∞) and their negative counterparts. The probability for each interval is p = 1/8 = .125 so the expected cell frequencies are E = np = (100) (0.125) = 12.5. The table of ranges and their corresponding frequencies is completed as follows.

Interval	Obs. Frequency.	Exp. Frequency.
$x \le 5332.5$	1	12.5
$5332.5 < x \le 5357.5$	4	12.5
$5357.5 < x \le 5382.5$	7	12.5
$5382.5 < x \le 5407.5$	24	12.5
$5407.5 < x \le 5432.5$	30	12.5
$5432.5 < x \le 5457.5$	20	12.5
$5457.5 < x \le 5482.5$	15	12.5
$x \ge 5482.5$	5	12.5

The test statistic is:

$$\chi_0^2 = \frac{(1-12.5)^2}{12.5} + \frac{(4-12.5)^2}{12.5} + \Lambda + \frac{(15-12.5)^2}{12.5} + \frac{(5-12.5)^2}{12.5} = 63.36$$

and we would reject if this value exceeds $\chi^2_{0.05,5} = 11.07$. Since $\chi_o^2 > \chi^2_{0.05,5}$, reject the hypothesis that the data are normally distributed

9-87 a) In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true mean overall distance for this brand of golf ball, μ.
2) $H_0 : \mu = 270$
3) $H_1 : \mu < 270$
4) $\alpha = 0.05$
5) Since n>>30 we can use the normal distribution

$$z_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

6) Reject H_0 if $z_0 < -z_\alpha$ where $z_{0.05} = 1.65$
7) $\bar{x} = 1.25$ s = 0.25 n = 20

$$z_0 = \frac{260.30 - 270.0}{13.41/\sqrt{100}} = -7.23$$

8) Since $-7.23 < -1.65$, reject the null hypothesis and conclude there is sufficient evidence to indicate that the true mean distance is less than 270 yds at $\alpha = 0.05$.

b) The P-value $\cong 0$.

c) We can divide the real line under a standard normal distribution into eight intervals with equal probability. These intervals are [0,.32), [0.32, 0.675), [0.675, 1.15), [1.15, ∞) and their negative counterparts. The probability for each interval is p = 1/8 = .125 so the expected cell frequencies are E = np = (100) (0.125) = 12.5. The table of ranges and their corresponding frequencies is completed as follows.

Interval	Obs. Frequency.	Exp. Frequency.
$x \le 244.88$	16	12.5
$244.88 < x \le 251.25$	6	12.5
$251.25 < x \le 256.01$	17	12.5
$256.01 < x \le 260.30$	9	12.5
$260.30 < x \le 264.59$	13	12.5
$264.59 < x \le 269.35$	8	12.5
$269.35 < x \le 275.72$	19	12.5
$x \ge 275.72$	12	12.5

The test statistic is:

$$\chi^2{}_o = \frac{(16-12.5)^2}{12.5} + \frac{(6-12.5)^2}{12.5} + \Lambda + \frac{(19-12.5)^2}{12.5} + \frac{(12-12.5)^2}{12.5} = 12$$

and we would reject if this value exceeds $\chi^2{}_{0.05,5} = 11.07$. Since it does, we can reject the hypothesis that the data are normally distributed.