Chapter 10 Selected Problem Solutions

Section 10-2

10-1. a) 1) The parameter of interest is the difference in fill volume, $\mu_1 - \mu_2$ (note that Δ_0=0)

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

6) Reject H_0 if $z_0 < -z_{\alpha/2} = -1.96$ or $z_0 > z_{\alpha/2} = 1.96$

7) $\bar{x}_1 = 16.015$ $\bar{x}_2 = 16.005$

$\sigma_1 = 0.02$ $\sigma_2 = 0.025$

$n_1 = 10$ $n_2 = 10$

$$z_0 = \frac{(16.015 - 16.005)}{\sqrt{\frac{(0.02)^2}{10} + \frac{(0.025)^2}{10}}} = 0.99$$

8) since -1.96 < 0.99 < 1.96, do not reject the null hypothesis and conclude there is no evidence that the two machine fill volumes differ at $\alpha = 0.05$.

b) *P-value* $= 2(1 - \Phi(0.99)) = 2(1 - 0.8389) = 0.3222$

c) Power = $1 - \beta$, where

$$\beta = \Phi\left(z_{\alpha/2} - \frac{\Delta - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right) - \Phi\left(-z_{\alpha/2} - \frac{\Delta - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right)$$

$$= \Phi\left(1.96 - \frac{0.04}{\sqrt{\frac{(0.02)^2}{10} + \frac{(0.025)^2}{10}}}\right) - \Phi\left(-1.96 - \frac{0.04}{\sqrt{\frac{(0.02)^2}{10} + \frac{(0.025)^2}{10}}}\right)$$

$$= \Phi(1.96 - 3.95) - \Phi(-1.96 - 3.95) = \Phi(-1.99) - \Phi(-5.91)$$

$= 0.0233 - 0$

$= 0.0233$

Power = 1 –0.0233 = 0.9967

d) $$(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(16.015 - 16.005) - 1.96\sqrt{\frac{(0.02)^2}{10} + \frac{(0.025)^2}{10}} \le \mu_1 - \mu_2 \le (16.015 - 16.005) + 1.96\sqrt{\frac{(0.02)^2}{10} + \frac{(0.025)^2}{10}}$$

$-0.0098 \le \mu_1 - \mu_2 \le 0.0298$

With 95% confidence, we believe the true difference in the mean fill volumes is between –0.0098 and 0.0298. Since 0 is contained in this interval, we can conclude there is no significant difference between the means.

e) Assume the sample sizes are to be equal, use $\alpha = 0.05$, $\beta = 0.05$, and $\Delta = 0.04$

$$n \cong \frac{(z_{\alpha/2}+z_\beta)^2(\sigma_1^2+\sigma_2^2)}{\delta^2} = \frac{(1.96+1.645)^2((0.02)^2+(0.025)^2)}{(0.04)^2} = 8.33, \quad n = 9,$$

use $n_1 = n_2 = 9$

10-5. $\bar{x}_1 = 30.87 \quad \bar{x}_2 = 30.68$

$\sigma_1 = 0.10 \quad \sigma_2 = 0.15$

$n_1 = 12 \quad n_2 = 10$

a) 90% two-sided confidence interval:

$$(\bar{x}_1-\bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}} \le \mu_1-\mu_2 \le (\bar{x}_1-\bar{x}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$

$$(30.87-30.68) - 1.645\sqrt{\frac{(0.10)^2}{12}+\frac{(0.15)^2}{10}} \le \mu_1-\mu_2 \le (30.87-30.68) + 1.645\sqrt{\frac{(0.10)^2}{12}+\frac{(0.15)^2}{10}}$$

$$0.0987 \le \mu_1-\mu_2 \le 0.2813$$

We are 90% confident that the mean fill volume for machine 1 exceeds that of machine 2 by between 0.0987 and 0.2813 fl. oz.

b) 95% two-sided confidence interval:

$$(\bar{x}_1-\bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}} \le \mu_1-\mu_2 \le (\bar{x}_1-\bar{x}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$

$$(30.87-30.68) - 1.96\sqrt{\frac{(0.10)^2}{12}+\frac{(0.15)^2}{10}} \le \mu_1-\mu_2 \le (30.87-30.68) + 1.96\sqrt{\frac{(0.10)^2}{12}+\frac{(0.15)^2}{10}}$$

$$0.0812 \le \mu_1-\mu_2 \le 0.299$$

We are 95% confident that the mean fill volume for machine 1 exceeds that of machine 2 by between 0.0812 and 0.299 fl. oz.

Comparison of parts a and b:

As the level of confidence increases, the interval width also increases (with all other values held constant).

c) 95% upper-sided confidence interval:

$$\mu_1-\mu_2 \le (\bar{x}_1-\bar{x}_2) + z_\alpha\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$

$$\mu_1-\mu_2 \le (30.87-30.68) + 1.645\sqrt{\frac{(0.10)^2}{12}+\frac{(0.15)^2}{10}}$$

$$\mu_1-\mu_2 \le 0.2813$$

With 95% confidence, we believe the fill volume for machine 1 exceeds the fill volume of machine 2 by no more than 0.2813 fl. oz.

10-7. $\bar{x}_1 = 89.6 \quad \bar{x}_2 = 92.5$

$\sigma_1^2 = 1.5 \quad \sigma_2^2 = 1.2$

$n_1 = 15 \quad n_2 = 20$

a) 95% confidence interval:

$$(\bar{x}_1-\bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}} \le \mu_1-\mu_2 \le (\bar{x}_1-\bar{x}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$

$$(89.6-92.5)-1.96\sqrt{\frac{1.5}{15}+\frac{1.2}{20}} \le \mu_1-\mu_2 \le (89.6-92.5)+1.96\sqrt{\frac{1.5}{15}+\frac{1.2}{20}}$$
$$-3.684 \le \mu_1-\mu_2 \le -2.116$$

With 95% confidence, we believe the mean road octane number for formulation 2 exceeds that of formulation 1 by between 2.116 and 3.684.

b) 1) The parameter of interest is the difference in mean road octane number, $\mu_1-\mu_2$ and $\Delta_0 = 0$

2) $H_0: \mu_1-\mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1-\mu_2 < 0$ or $\mu_1 < \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$z_0 = \frac{(\bar{x}_1-\bar{x}_2)-\Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$$

6) Reject H_0 if $z_0 < -z_\alpha = -1.645$

7) $\bar{x}_1 = 89.6 \quad \bar{x}_2 = 92.5$

$\sigma_1^2 = 1.5 \quad \sigma_2^2 = 1.2$

$n_1 = 15 \quad n_2 = 20$

$$z_0 = \frac{(89.6-92.5)-0}{\sqrt{\frac{(1.5)^2}{15}+\frac{(1.2)^2}{20}}} = -7.254$$

8) Since $-7.25 < -1.645$ reject the null hypothesis and conclude the mean road octane number for formulation 2 exceeds that of formulation 1 using $\alpha = 0.05$.

c) P-value = $P(z \le -7.25) = 1-P(z \le 7.25) = 1-1 \cong 0$

10-9. 95% level of confidence, $E = 1$, and $z_{0.025} = 1.96$

$$n \cong \left(\frac{z_{0.025}}{E}\right)^2(\sigma_1^2+\sigma_2^2) = \left(\frac{1.96}{1}\right)^2(1.5+1.2) = 10.37,$$ $n = 11$, use $n_1 = n_2 = 11$

10-11.

Catalyst 1	Catalyst 2
$\bar{x}_1 = 65.22$	$\bar{x}_2 = 68.42$
$\sigma_1 = 3$	$\sigma_2 = 3$
$n_1 = 10$	$n_2 = 10$

a) 95% confidence interval on $\mu_1-\mu_2$, the difference in mean active concentration

$$(\bar{x}_1-\bar{x}_2)-z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}} \le \mu_1-\mu_2 \le (\bar{x}_1-\bar{x}_2)+z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$
$$(65.22-68.42)-1.96\sqrt{\frac{(3)^2}{10}+\frac{(3)^2}{10}} \le \mu_1-\mu_2 \le (65.22-68.42)+1.96\sqrt{\frac{(3)^2}{10}+\frac{(3)^2}{10}}$$
$$-5.83 \le \mu_1-\mu_2 \le -0.57$$

We are 95% confident that the mean active concentration of catalyst 2 exceeds that of catalyst 1 by between 0.57 and 5.83 g/l.

b) Yes, since the 95% confidence interval did not contain the value 0, we would conclude that the mean active concentration depends on the choice of catalyst.

10-13. 1) The parameter of interest is the difference in mean active concentration, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

6) Reject H_0 if $z_0 < -z_{\alpha/2} = -1.96$ or $z_0 > z_{\alpha/2} = 1.96$

7) $\bar{x}_1 = 750.2$ $\bar{x}_2 = 756.88$ $\delta = 0$

$\sigma_1 = 20$ $\sigma_2 = 20$

$n_1 = 15$ $n_2 = 8$

$$z_0 = \frac{(750.2 - 756.88) - 0}{\sqrt{\frac{(20)^2}{15} + \frac{(20)^2}{8}}} = -2.385$$

8) Since $-2.385 < -1.96$ reject the null hypothesis and conclude the mean active concentrations do differ significantly at $\alpha = 0.05$.

P-value $= 2(1 - \Phi(2.385)) = 2(1 - 0.99146) = 0.0171$

The conclusions reached by the confidence interval of the previous problem and the test of hypothesis conducted here are the same. A two-sided confidence interval can be thought of as representing the "acceptance region" of a hypothesis test, given that the level of significance is the same for both procedures. Thus if the value of the parameter under test that is specified in the null hypothesis falls outside the confidence interval, this is equivalent to rejecting the null hypothesis.

Section 10-3

10-17 a) 1) The parameter of interest is the difference in mean rod diameter, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{\alpha/2, n_1+n_2-2}$ where $-t_{0.025,30} = -2.042$ or $t_0 > t_{\alpha/2, n_1+n_2-2}$ where $t_{0.025,30} = 2.042$

7)) $\bar{x}_1 = 8.73$ $\bar{x}_2 = 8.68$

$s_1^2 = 0.35$ $s_2^2 = 0.40$

$n_1 = 15$ $n_2 = 17$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{14(0.35) + 16(0.40)}{30}} = 0.614$$

$$t_0 = \frac{(8.73 - 8.68)}{0.614\sqrt{\frac{1}{15} + \frac{1}{17}}} = 0.230$$

8) Since $-2.042 < 0.230 < 2.042$, do not reject the null hypothesis and conclude the two machines do not produce rods with significantly different mean diameters at $\alpha = 0.05$.

b) P-value = $2P(t > 0.230) > 2(0.40)$, P-value > 0.80

c) 95% confidence interval: $t_{0.025,30} = 2.042$

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$(8.73 - 8.68) - 2.042(0.614)\sqrt{\frac{1}{15}+\frac{1}{17}} \le \mu_1 - \mu_2 \le (8.73 - 8.68) + 2.042(0.643)\sqrt{\frac{1}{15}+\frac{1}{17}}$$

$$-0.394 \le \mu_1 - \mu_2 \le 0.494$$

Since zero is contained in this interval, we are 95% confident that machine 1 and machine 2 do not produce rods whose diameters are significantly different.

10-21. a) 1) The parameter of interest is the difference in mean etch rate, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \ne 0$ or $\mu_1 \ne \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{\alpha/2,n_1+n_2-2}$ where $-t_{0.025,18} = -2.101$ or $t_0 > t_{\alpha/2,n_1+n_2-2}$ where $t_{0.025,18} = 2.101$

7) $\bar{x}_1 = 9.97$ $\bar{x}_2 = 10.4$

$s_1 = 0.422$ $s_2 = 0.231$

$n_1 = 10$ $n_2 = 10$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{9(0.422)^2 + 9(0.231)^2}{18}} = 0.340$$

$$t_0 = \frac{(9.97 - 10.4)}{0.340\sqrt{\frac{1}{10}+\frac{1}{10}}} = -2.83$$

8) Since $-2.83 < -2.101$ reject the null hypothesis and conclude the two machines mean etch rates do significantly differ at $\alpha = 0.05$.

b) P-value = $2P(t < -2.83)$ $2(0.005) <$ P-value $< 2(0.010) = 0.010 <$ P-value < 0.020

c) 95% confidence interval: $t_{0.025,18} = 2.101$

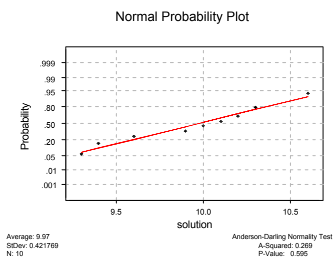
$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

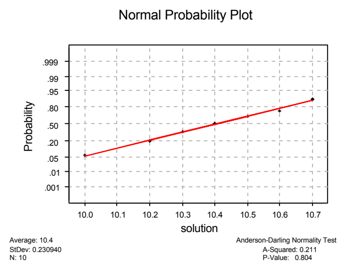
$$(9.97 - 10.4) - 2.101(.340)\sqrt{\frac{1}{10}+\frac{1}{10}} \le \mu_1 - \mu_2 \le (9.97 - 10.4) + 2.101(.340)\sqrt{\frac{1}{10}+\frac{1}{10}}$$

$$-0.749 \le \mu_1 - \mu_2 \le -0.111$$

We are 95% confident that the mean etch rate for solution 2 exceeds the mean etch rate for solution 1 by between 0.1105 and 0.749.

d) According to the normal probability plots, the assumption of normality appears to be met since the data from both samples fall approximately along straight lines. The equality of variances does not appear to be severely violated either since the slopes are approximately the same for both samples.





10-27 a) 1) The parameter of interest is the difference in mean wear amount, $\mu_1 - \mu_2$.

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{0.025,27}$ where $-t_{0.025,27} = -2.052$ or $t_0 > t_{0.025,27}$ where $t_{0.025,27} = 2.052$ since

$$v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} = 26.98$$

$$v \cong 26$$

(truncated)

7) $\bar{x}_1 = 20$ $\bar{x}_2 = 15$ $\Delta_0 = 0$

$s_1 = 2$ $s_2 = 8$

$n_1 = 25$ $n_2 = 25$

$$t_0 = \frac{(20 - 15) - 0}{\sqrt{\frac{(2)^2}{25} + \frac{(8)^2}{25}}} = 3.03$$

8) Since 3.03 > 2.056 reject the null hypothesis and conclude that the data support the claim that the two companies produce material with significantly different wear at the 0.05 level of significance.

b) P-value = 2P(t > 3.03), 2(0.0025) < P-value < 2(0.005)

0.005 < P-value < 0.010

c) 1) The parameter of interest is the difference in mean wear amount, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$

3) $H_1: \mu_1 - \mu_2 > 0$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\overline{x}_1 - \overline{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

6) Reject the null hypothesis if $t_0 > t_{0.05,27}$ where $t_{0.05,26} = 1.706$ since

7) $\overline{x}_1 = 20 \quad \overline{x}_2 = 15$

$s_1 = 2 \quad s_2 = 8 \quad \Delta_0 = 0$

$n_1 = 25 \quad n_2 = 25$

$$t_0 = \frac{(20-15)-0}{\sqrt{\frac{(2)^2}{25} + \frac{(8)^2}{25}}} = 3.03$$

8) Since 3.03 > 1.706 reject the null hypothesis and conclude that the data support the claim that the material from company 1 has a higher mean wear than the material from company 2 using a 0.05 level of significance.

10-29. If $\alpha = 0.01$, construct a 99% lower one-sided confidence interval on the difference to answer question 10-28.
$t_{0.005,19} = 2.878$

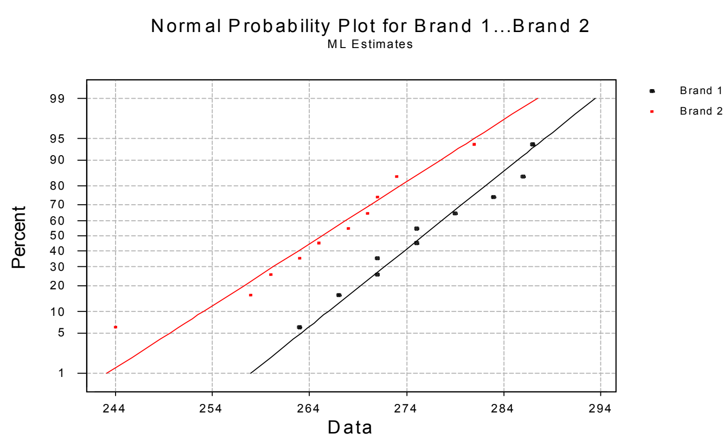
$$(\overline{x}_1 - \overline{x}_2) - t_{\alpha/2,v}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \le \mu_1 - \mu_2 \le (\overline{x}_1 - \overline{x}_2) + t_{\alpha/2,v}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(103.5 - 99.7) - 2.878\sqrt{\frac{(10.2)^2}{12} + \frac{(20.1)^2}{13}} \le \mu_1 - \mu_2 \le (103.5 - 99.7) - 2.878\sqrt{\frac{(10.2)^2}{12} + \frac{(20.1)^2}{13}}$$

$$-14.34 \le \mu_1 - \mu_2 \le 21.94.$$

Since the interval contains 0, we are 99% confident there is no difference in the mean coating thickness between the two temperatures; that is, raising the process temperature does not significantly reduce the mean coating thickness.

10-31 a.)



b . 1) The parameter of interest is the difference in mean overall distance, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$
3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$
4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{\alpha/2,n_1+n_2-2}$ where $-t_{0.025,18} = -2.101$ or $t_0 > t_{\alpha/2,n_1+n_2-2}$ where $t_{0.025,18} = 2.101$

7) $\bar{x}_1 = 275.7$ $\quad \bar{x}_2 = 265.3$ $\quad s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$

$s_1 = 8.03$ $\quad s_2 = 10.04$ $\quad = \sqrt{\frac{9(8.03)^2 + 9(10.04)^2}{20}} = 9.09$

$n_1 = 10$ $\quad n_2 = 10$

$$t_0 = \frac{(275.7 - 265.3)}{9.09\sqrt{\frac{1}{10} + \frac{1}{10}}} = 2.558$$

8) Since 2.558>2.101 reject the null hypothesis and conclude that the data do not support the claim that both brands have the same mean overall distance at $\alpha = 0.05$. It appears that brand 1 has the higher mean differnce.

c.)P-value = 2P$(t < 2.558)$ P-value ≈ 2(0.01)=0.02

d.) $d = \frac{5}{2(9.09)}0.275$ $\quad \beta$=0.95 $\quad$ Power =1-0.95=0.05

e.) 1-β=0..75 $\quad \beta$=0..27 $\quad d = \frac{3}{2(9.09)} = 0.165$ $\quad n^*$=100 $\quad n = \frac{100+1}{2} = 50.5$

Therefore, n=51

f.) $$(\bar{x}_1 - \bar{x}_2) - t_{\alpha,v}s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \leq \mu_1 - \mu_2 \leq (\bar{x}_1 - \bar{x}_2) + t_{\alpha,v}s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$(275.7 - 265.3) - 2.101(9.09)\sqrt{\frac{1}{10} + \frac{1}{10}} \leq \mu_1 - \mu_2 \leq (275.7 - 265.3) + 2.101(9.09)\sqrt{\frac{1}{10} + \frac{1}{10}}$$

$$1.86 \leq \mu_1 - \mu_2 \leq 18.94$$

Section 10-4

10-37 $\bar{d} = 868.375$ $\quad s_d = 1290$, n = 8 $\quad$ where d_i = brand 1 - brand 2
99% confidence interval:

$$\bar{d} - t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right) \leq \mu_d \leq \bar{d} + t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right)$$

$$868.375 - 3.499\left(\frac{1290}{\sqrt{8}}\right) \le \mu_d \le 868.375 + 3.499\left(\frac{1290}{\sqrt{8}}\right)$$

$$-727.46 \le \mu_d \le 2464.21$$

Since this confidence interval contains zero, we are 99% confident there is no significant difference between the two brands of tire.

10-39. 1) The parameter of interest is the difference in blood cholesterol level, μ_d where d_i = Before − After.

2) $H_0 : \mu_d = 0$

3) $H_1 : \mu_d > 0$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

6) Reject the null hypothesis if $t_0 > t_{0.05,14}$ where $t_{0.05,14} = 1.761$

7) $\bar{d} = 26.867$
$s_d = 19.04$
$n = 15$

$$t_0 = \frac{26.867}{19.04 / \sqrt{15}} = 5.465$$

8) Since 5.465 > 1.761 reject the null and conclude the data support the claim that the mean difference in cholesterol levels is significantly less after fat diet and aerobic exercise program at the 0.05 level of significance.

Section 10-5

10-47. 1) The parameters of interest are the variances of concentration, σ_1^2, σ_2^2

2) $H_0 : \sigma_1^2 = \sigma_2^2$

3) $H_1 : \sigma_1^2 \ne \sigma_2^2$

4) $\alpha = 0.05$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject the null hypothesis if $f_0 < f_{0.975,9,15}$ where $f_{0.975,9,15} = 0.265$ or $f_0 > f_{0.025,9,15}$ where $f_{0.025,9,15} = 3.12$

7) $n_1 = 10$ $n_2 = 16$
$s_1 = 4.7$ $s_2 = 5.8$

$$f_0 = \frac{(4.7)^2}{(5.8)^2} = 0.657$$

8) Since 0.265 < 0.657 < 3.12 do not reject the null hypothesis and conclude there is insufficient evidence to indicate the two population variances differ significantly at the 0.05 level of significance.

10-51 a) 90% confidence interval for the ratio of variances:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2, n_1-1, n_2-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2, n_1-1, n_2-1}$$

$$\left(\frac{(0.6)^2}{(0.8)^2}\right) 0.156 \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{(0.6)^2}{(0.8)^2}\right) 6.39$$

$$0.08775 \le \frac{\sigma_1^2}{\sigma_2^2} \le 3.594$$

b) 95% confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2,n_1-1,n_2-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2,n_1-1,n_2-1}$$

$$\left(\frac{(0.6)^2}{(0.8)^2}\right) 0.104 \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{(0.6)^2}{(0.8)^2}\right) 9.60$$

$$0.0585 \le \frac{\sigma_1^2}{\sigma_2^2} \le 5.4$$

The 95% confidence interval is wider than the 90% confidence interval.

c) 90% lower-sided confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha,n_1-1,n_2-1} \le \frac{\sigma_1^2}{\sigma_2^2}$$

$$\left(\frac{(0.6)^2}{(0.8)^2}\right) 0.243 \le \frac{\sigma_1^2}{\sigma_2^2}$$

$$0.137 \le \frac{\sigma_1^2}{\sigma_2^2}$$

10-55 1) The parameters of interest are the thickness variances, σ_1^2, σ_2^2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \ne \sigma_2^2$

4) $\alpha = 0.01$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject the null hypothesis if $f_0 < f_{0.995,10,12}$ where $f_{0.995,10,12}$ =0.1766 or $f_0 > f_{0.005,10,12}$ where $f_{0.005,10,12} = 2.91$

7) $n_1 = 11$ $\quad n_2 = 13$

$s_1 = 10.2$ $\quad s_2 = 20.1$

$$f_0 = \frac{(10.2)^2}{(20.1)^2} = 0.2575$$

8) Since 0.1766 >0.2575 > 5.0855 do not reject the null hypothesis and conclude the thickness variances are not equal at the 0.01 level of significance.

10-59 1) The parameters of interest are the overall distance standard deviations, σ_1, σ_2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \ne \sigma_2^2$

4) $\alpha = 0.05$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject the null hypothesis if $f_0 < f_{.0.975,9,9} = 0.248$ or $f_0 > f_{0.025,9,9} = 4.03$

7) $n_1 = 10$ $n_2 = 10$ $s_1 = 8.03$ $s_2 = 10.04$

$$f_0 = \frac{(8.03)^2}{(10.04)^2} = 0.640$$

8) Since 0.248 < 0.640 < 4.04 do not reject the null hypothesis and conclude there is no evidence to support the claim that there is a difference in the standard deviation of the overall distance of the two brands at the 0.05 level of significance.

95% confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2,n_1-1,n_2-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2,n_1-1,n_2-1}$$

$$(0.640)0.248 \le \frac{\sigma_1^2}{\sigma_2^2} \le (0.640)4.03$$

$$0.159 \le \frac{\sigma_1^2}{\sigma_2^2} \le 2.579$$

Since the value 1 is contained within this interval, we are 95% confident there is no significant difference in the standard deviation of the overall distance of the two brands at the 0.05 level of significance.

<u>Section 10-6</u>

10-61. 1) the parameters of interest are the proportion of defective parts, p_1 and p_2

2) $H_0 : p_1 = p_2$

3) $H_1 : p_1 \ne p_2$

4) $\alpha = 0.05$

5) Test statistic is

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad \text{where}$$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

6) Reject the null hypothesis if $z_0 < -z_{0.025}$ where $-z_{0.025} = -1.96$ or $z_0 > z_{0.025}$ where $z_{0.025} = 1.96$

7) $n_1 = 300$ $n_2 = 300$
$x_1 = 15$ $x_2 = 8$

$\hat{p}_1 = 0.05$ $\hat{p}_2 = 0.0267$ $\hat{p} = \frac{15+8}{300+300} = 0.0383$

$$z_0 = \frac{0.05 - 0.0267}{\sqrt{0.0383(1-0.0383)\left(\frac{1}{300} + \frac{1}{300}\right)}} = 1.49$$

8) Since −1.96 < 1.49 < 1.96 do not reject the null hypothesis and conclude that yes the evidence indicates that there is not a significant difference in the fraction of defective parts produced by the two machines

at the 0.05 level of significance.

P-value = 2(1–P(z < 1.49)) = 0.13622

10-63. a) Power = 1 – β

$$\beta = \Phi\left(\frac{z_{\alpha/2}\sqrt{\overline{p}\overline{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}-(p_1-p_2)}{\hat{\sigma}_{\hat{p}_1-\hat{p}_2}}\right)-\Phi\left(\frac{-z_{\alpha/2}\sqrt{\overline{p}\overline{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}-(p_1-p_2)}{\hat{\sigma}_{\hat{p}_1-\hat{p}_2}}\right)$$

$$\overline{p}=\frac{300(0.05)+300(0.01)}{300+300}=0.03 \qquad \overline{q}=0.97$$

$$\hat{\sigma}_{\hat{p}_1-\hat{p}_2}=\sqrt{\frac{0.05(1-0.05)}{300}+\frac{0.01(1-0.01)}{300}}=0.014$$

$$\beta = \Phi\left(\frac{1.96\sqrt{0.03(0.97)\left(\frac{1}{300}+\frac{1}{300}\right)}-(0.05-0.01)}{0.014}\right)-\Phi\left(\frac{-1.96\sqrt{0.03(0.97)\left(\frac{1}{300}+\frac{1}{300}\right)}-(0.05-0.01)}{0.014}\right)$$

$$=\Phi(-0.91)-\Phi(-4.81)=0.18141-0=0.18141$$

Power = 1 – 0.18141 = 0.81859

b) $$n=\frac{\left(z_{\alpha/2}\sqrt{\frac{(p_1+p_2)(q_1+q_2)}{2}}+z_\beta\sqrt{p_1q_1+p_2q_2}\right)^2}{(p_1-p_2)^2}$$

$$=\frac{\left(1.96\sqrt{\frac{(0.05+0.01)(0.95+0.99)}{2}}+1.29\sqrt{0.05(0.95)+0.01(0.99)}\right)^2}{(0.05-0.01)^2}=382.11$$

n = 383

10-67 95% confidence interval on the difference:

$$(\hat{p}_1-\hat{p}_2)-z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}\le p_1-p_2\le(\hat{p}_1-\hat{p}_2)+z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

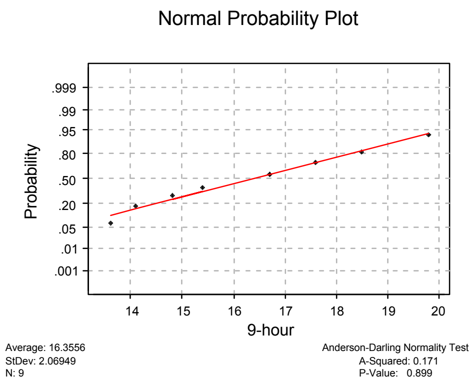
$$(0.77-0.6675)-1.96\sqrt{\frac{0.77(1-0.77)}{500}+\frac{0.6675(1-0.6675)}{400}}\le p_1-p_2\le(0.77-0.6675)+1.96\sqrt{\frac{0.77(1-0.77)}{500}+\frac{0.6675(1-0.6675)}{400}}$$

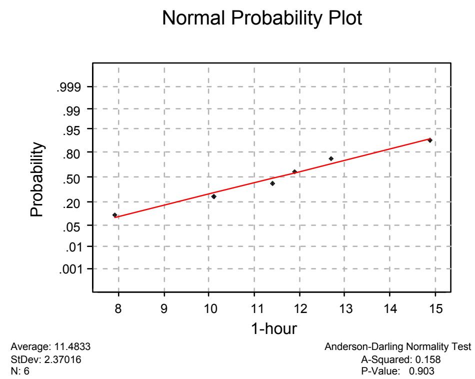
$$0.0434\le p_1-p_2\le 0.1616$$

Since this interval does not contain the value zero, we are 95% confident there is a significant difference in the proportions of support for increasing the speed limit between residents of the two counties and that the difference in proportions is between 0.0434 and 0.1616.

Supplemental Exercises

10-69 a) Assumptions that must be met are normality, equality of variance, independence of the observations and of the populations. Normality and equality of variances appears to be reasonable, see normal probability plot. The data appear to fall along a straight line and the slopes appear to be the same. Independence of the observations for each sample is assumed. It is also reasonable to assume that the two populations are independent.





b) $\bar{x}_1 = 16.36$ $\quad \bar{x}_2 = 11.486$

$s_1 = 2.07$ $\quad s_2 = 2.37$

$n_1 = 9$ $\quad n_2 = 6$

99% confidence interval: $t_{\alpha/2,n_1+n_2-2} = t_{0.005,13}$ where $t_{0.005,13} = 3.012$

$$s_p = \sqrt{\frac{8(2.07)^2 + 5(2.37)^2}{13}} = 2.19$$

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$(16.36 - 11.486) - 3.012(2.19)\sqrt{\frac{1}{9} + \frac{1}{6}} \le \mu_1 - \mu_2 \le (16.36 - 11.486) + 3.012(2.19)\sqrt{\frac{1}{9} + \frac{1}{6}}$$

$1.40 \le \mu_1 - \mu_2 \le 8.36$

c) Yes, we are 99% confident the results from the first test condition exceed the results of the second test condition by between 1.40 and 8.36 ($\times 10^6$ PA).

10-73 a) 1) The parameters of interest are the proportions of children who contract polio, p_1 , p_2

2) $H_0 : p_1 = p_2$

3) $H_1 : p_1 \neq p_2$

4) $\alpha = 0.05$

5) The test statistic is

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{\alpha/2} = 1.96$

7) $\hat{p}_1 = \frac{x_1}{n_1} = \frac{110}{201299} = 0.00055$ (Placebo) $\quad \hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = 0.000356$

$\hat{p}_2 = \frac{x_2}{n_2} = \frac{33}{200745} = 0.00016$ (Vaccine)

$$z_0 = \frac{0.00055 - 0.00016}{\sqrt{0.000356(1 - 0.000356)\left(\frac{1}{201299} + \frac{1}{200745}\right)}} = 6.55$$

8) Since 6.55 > 1.96 reject H_0 and conclude the proportion of children who contracted polio is significantly different at $\alpha = 0.05$.

b) $\alpha = 0.01$ Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{\alpha/2}$ =2.33

$z_0 = 6.55$

Since 6.55 > 2.33, reject H_0 and conclude the proportion of children who contracted polio is different at $\alpha = 0.01$.

c) The conclusions are the same since z_0 is so large it exceeds $z_{\alpha/2}$ in both cases.

10-79.

$$n = \frac{\left(2.575\sqrt{\frac{(0.9+0.6)(0.1+0.4)}{2}} + 1.28\sqrt{0.9(0.1)+0.6(0.4)}\right)^2}{(0.9-0.6)^2}$$

$$= \frac{5.346}{0.09} = 59.4$$

$$n = 60$$

10-81. $H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

$n_1 = n_2 = n$

$\beta = 0.10$

$\alpha = 0.05$

Assume normal distribution and $\sigma_1^2 = \sigma_2^2 = \sigma^2$

$$\mu_1 = \mu_2 + \sigma$$

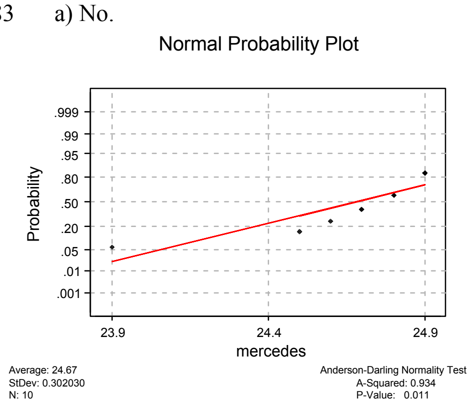
$$d = \frac{|\mu_1 - \mu_2|}{2\sigma} = \frac{\sigma}{2\sigma} = \frac{1}{2}$$

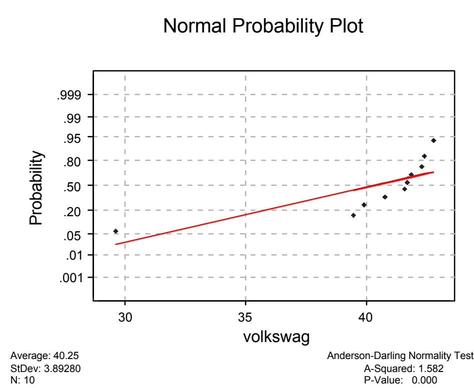
From Chart VI (e), $n^* = 50$

$$n = \frac{n^* + 1}{2} = \frac{50+1}{2} = 25.5$$

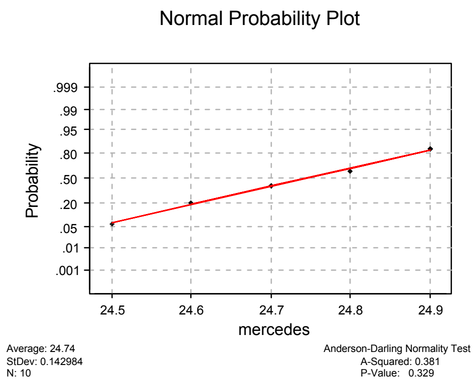
$n_1 = n_2 = 26$

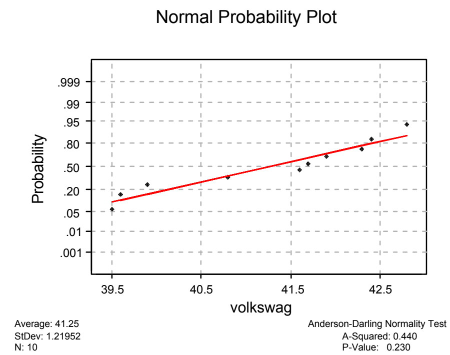
10-83 a) No.





b) The normal probability plots indicate that the data follow normal distributions since the data appear to fall along a straight line. The plots also indicate that the variances could be equal since the slopes appear to be the same.





c) By correcting the data points, it is more apparent the data follow normal distributions. Note that one unusual observation can cause an analyst to reject the normality assumption.

d) 95% confidence interval on the ratio of the variances, σ_V^2 / σ_M^2

$$s_V^2 = 1.49 \qquad f_{9,9,0.025} = 4.03$$

$$s_M^2 = 0.0204 \qquad f_{9,9,0.975} = \frac{1}{f_{9,9,0.025}} = \frac{1}{4.03} = 0.248$$

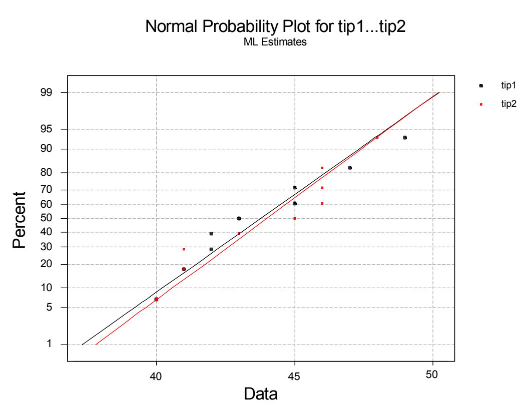
$$\left(\frac{s_V^2}{s_M^2}\right) f_{9,9,0.975} < \frac{\sigma_V^2}{\sigma_M^2} < \left(\frac{s_V^2}{s_M^2}\right) f_{9,9,0.025}$$

$$\left(\frac{1.49}{0.0204}\right) 0.248 < \frac{\sigma_V^2}{\sigma_M^2} < \left(\frac{1.49}{0.0204}\right) 4.03$$

$$18.124 < \frac{\sigma_V^2}{\sigma_M^2} < 294.35$$

Since the does not include the value of unity, we are 95% confident that there is evidence to reject the claim that the variability in mileage performance is different for the two types of vehicles. There is evidence that the variability is greater for a Volkswagen than for a Mercedes.

10-85 a) Underlying distributions appear to be normal since the data fall along a straight line on the normal probability plots. The slopes appear to be similar, so it is reasonable to assume that $\sigma_1^2 = \sigma_2^2$.



b) 1) The parameter of interest is the difference in mean volumes, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \delta}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) Reject H_0 if $t_0 < -t_{\alpha/2,v}$ or $z_0 > t_{\alpha/2,v}$ where $t_{\alpha/2,v} = t_{0.025,18} = 2.101$

7) $\bar{x}_1 = 752.7$ $\quad\bar{x}_2 = 755.6$ $\quad s_p = \sqrt{\frac{9(1.252)^2 + 9(0.843)^2}{18}} = 1.07$

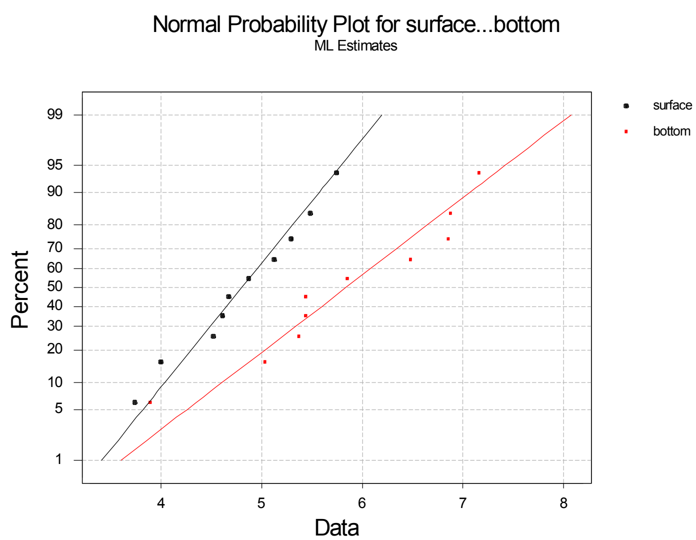
$s_1 = 1.252$ $\quad s_2 = 0.843$

$n_1 = 10$ $\quad n_2 = 10$

$$t_0 = \frac{(752.7 - 755.6) - 0}{1.07\sqrt{\frac{1}{10} + \frac{1}{10}}} = -6.06$$

8) Since $-6.06 < -2.101$, reject H_0 and conclude there is a significant difference between the two wineries with respect to the mean fill volumes.

10-89 a.) The data from both depths appear to be normally distributed, but the slopes are not equal. Therefore, it may not be assumed that $\sigma_1^2 = \sigma_2^2$.



b.) 1) The parameter of interest is the difference in mean HCB concentration, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{0.025,15}$ where $-t_{0.025,15} = -2.131$ or $t_0 > t_{0.025,15}$ where $t_{0.025,15} = 2.131$ since

$$v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} = 15.06$$

$$v \cong 15$$

(truncated)

7) $\overline{x}_1 = 4.804$ $\overline{x}_2 = 5.839$ $s_1 = 0.631$ $s_2 = 1.014$

$n_1 = 10$ $n_2 = 10$

$$t_0 = \frac{(4.804 - 5.839)}{\sqrt{\frac{(0.631)^2}{10} + \frac{(1.014)^2}{10}}} = -2.74$$

8) Since –2.74 < -2.131 reject the null hypothesis and conclude that the data support the claim that the mean HCB concentration is different at the two depths sampled at the 0.05 level of significance.

b) P-value = 2P(t < -2.74), 2(0.005) < P-value < 2(0.01)

0.001 < P-value < 0.02

c) $\Delta = 2$ $\alpha = 0.05$ $n_1 = n_2 = 10$ $d = \frac{2}{2(1)} = 1$

From Chart VI (e) we find $\beta = 0.20$, and then calculate Power = 1- β = 0.80

d.) $\Delta = 2$ $\alpha = 0.05$ $d = \frac{2}{2(1)} = 0.5$, $\beta = 0.0$

From Chart VI (e) we find n*=50 and $n = \frac{50+1}{2} = 25.5$, so n=26