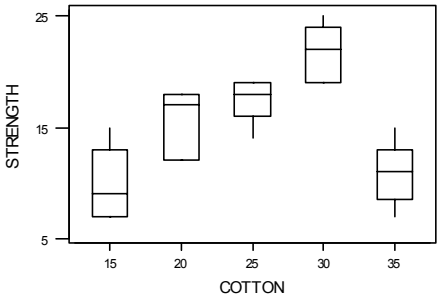
Chapter 13 Selected Problem Solutions

Section 13-2

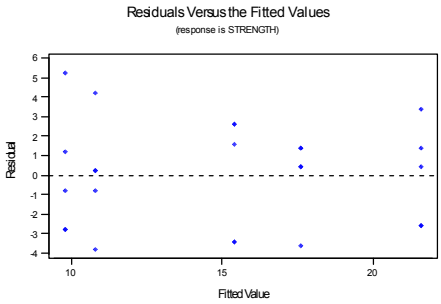
13-1. a) Analysis of Variance for STRENGTH

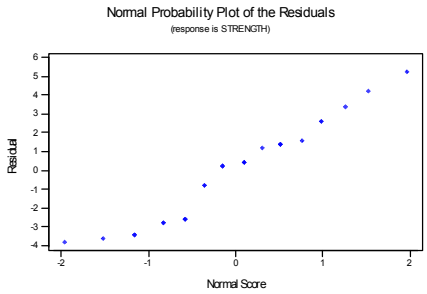
Source	DF	SS	MS	F	P
COTTON	4	475.76	118.94	14.76	0.000
Error	20	161.20	8.06		
Total	24	636.96			

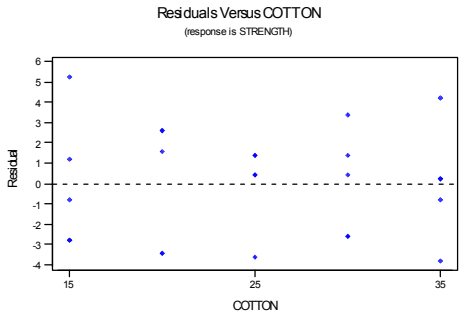
Reject H_0 and conclude that cotton percentage affects mean breaking strength.
b) Tensile strength seems to increase to 30% cotton and declines at 35% cotton.



c) The normal probability plot and the residual plots show that the model assumptions are reasonable.







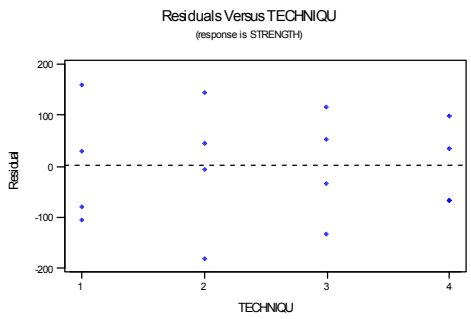
13-3. a) Analysis of Variance for STRENGTH

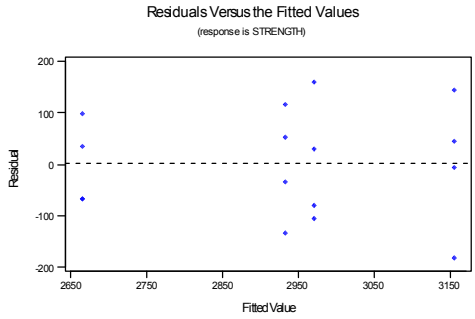
Source	DF	SS	MS	F	P
TECHNIQU	3	489740	163247	12.73	0.000
Error	12	153908	12826		
Total	15	643648			

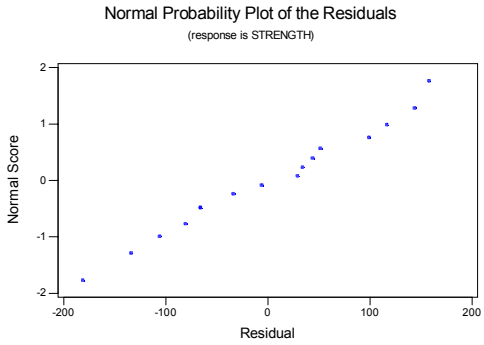
Reject H_0. Techniques affect the mean strength of the concrete.

b) *P-value* ≅ 0

c) Residuals are acceptable







13-5. a)

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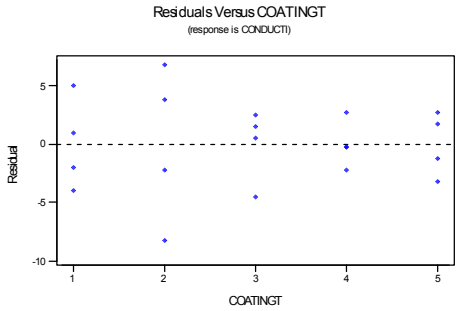
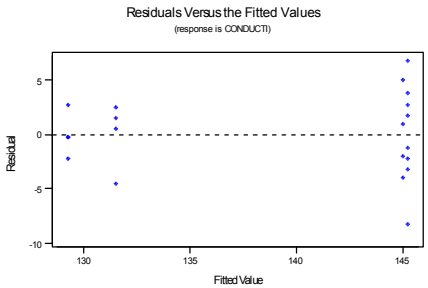
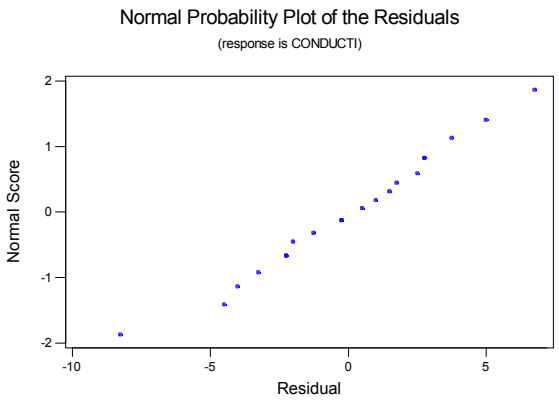
Analysis of Variance for CONDUCTIVITY
Source          DF        SS        MS        F        P
COATINGTYPE      4    1060.5     265.1    16.35    0.000
Error           15     243.3      16.2
Total           19    1303.8
```

Reject H_0 ; P-value ≅ 0.

b) There is some indication that the variability of the response may be increasing as the mean response increases. There appears to be an outlier on the normal probability plot.







c) 95% Confidence interval on the mean of coating type 1.

$$\bar{y}_1 - t_{0.025,15}\sqrt{\frac{MS_E}{n}} \le \mu_i \le \bar{y}_1 + t_{0.015,15}\sqrt{\frac{MS_E}{n}}$$

$$145.00 - 2.131\sqrt{\frac{16.2}{4}} \le \mu_1 \le 145.00 + 2.131\sqrt{\frac{16.2}{4}}$$

$$140.71 \le \mu_1 \le 149.29$$

d.) 99% confidence interval on the difference between the means of coating types 1 and 4.

$$\bar{y}_1 - \bar{y}_4 - t_{0.005,15}\sqrt{\frac{2MS_E}{n}} \le \mu_1 - \mu_4 \le \bar{y}_1 - \bar{y}_4 + t_{0.005,15}\sqrt{\frac{2MS_E}{n}}$$

$$(145.00 - 129.25) - 2.947\sqrt{\frac{2(16.2)}{4}} \le \mu_1 - \mu_4 \le (145.00 - 129.25) - 2.947\sqrt{\frac{2(16.2)}{4}}$$

$$7.36 \le \mu_1 - \mu_4 \le 24.14$$

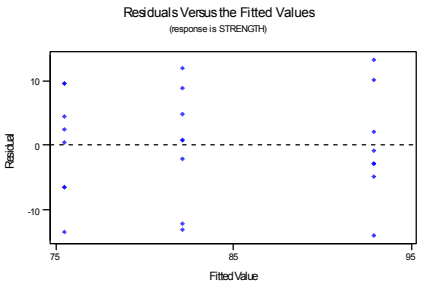
13-9. a) Analysis of Variance for STRENGTH

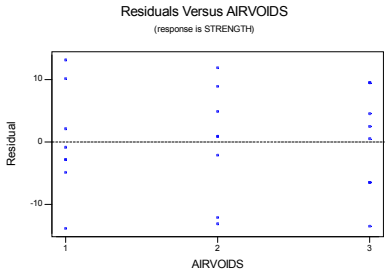
Source	DF	SS	MS	F	P
AIRVOIDS	2	1230.3	615.1	8.30	0.002
Error	21	1555.8	74.1		
Total	23	2786.0			

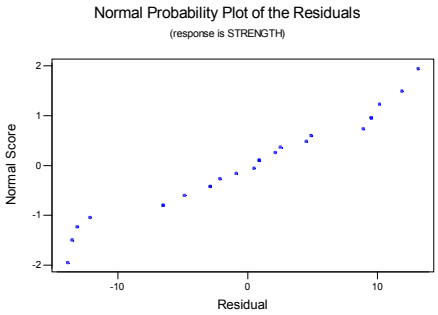
Reject H_0

b) *P-value* = 0.002

c) The residual plots show that the assumptions of equality of variance is reasonable. The normal probability plot has some curvature in the tails.







d) 95% Confidence interval on the mean of retained strength where there is a high level of air voids

$$\bar{y}_3 - t_{0.025,21}\sqrt{\frac{MS_E}{n}} \le \mu_i \le \bar{y}_3 + t_{0.015,21}\sqrt{\frac{MS_E}{n}}$$

$$8.229 - 2.080\sqrt{\frac{74.1}{8}} \le \mu_3 \le 8.229 + 2.080\sqrt{\frac{74.1}{8}}$$

$$69.17 \le \mu_1 \le 81.83$$

e) 95% confidence interval on the difference between the means of retained strength at the high level and the low levels of air voids.

$$\bar{y}_1 - \bar{y}_3 - t_{0.025,21}\sqrt{\frac{2MS_E}{n}} \le \mu_1 - \mu_3 \le \bar{y}_1 - \bar{y}_3 + t_{0.025,21}\sqrt{\frac{2MS_E}{n}}$$

$$(92.875 - 75.5) - 2.080\sqrt{\frac{2(74.1)}{8}} \le \mu_1 - \mu_4 \le (92.875 - 75.5) - 2.080\sqrt{\frac{2(74.1)}{8}}$$

$$8.42 \le \mu_1 - \mu_4 \le 26.38$$

Section 13-3

13-21 a) Analysis of Variance for OUTPUT

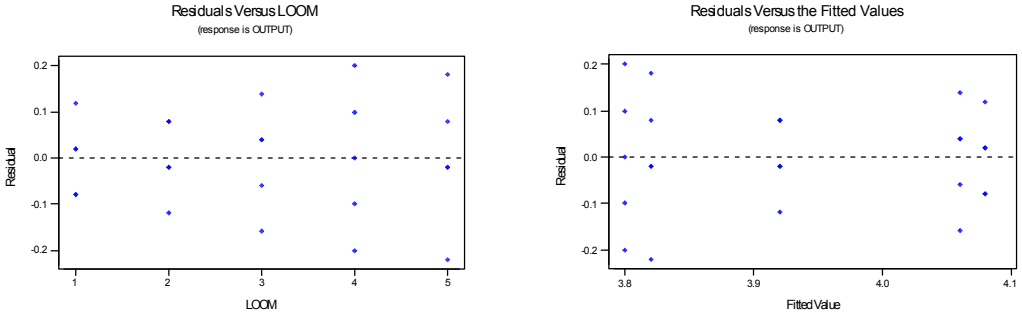
Source	DF	SS	MS	F	P
LOOM	4	0.3416	0.0854	5.77	0.003
Error	20	0.2960	0.0148		
Total	24	0.6376			

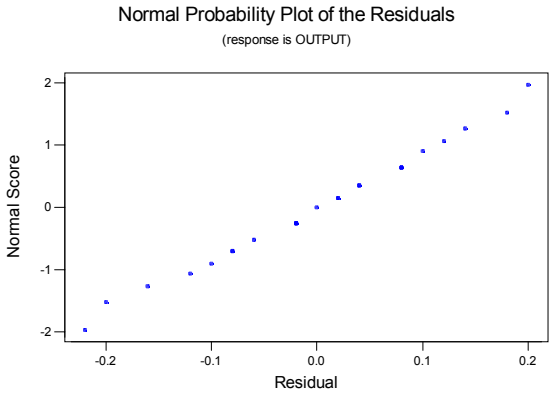
Reject H_0, and conclude that there are significant differences among the looms.

b) $\hat{\sigma}_\tau^2 = \dfrac{MS_{Treatments} - MS_E}{n} = \dfrac{0.0854 - 0.0148}{5} = 0.01412$

c) $\hat{\sigma}^2 = MS_E = 0.0148$

d) Residuals plots are acceptable



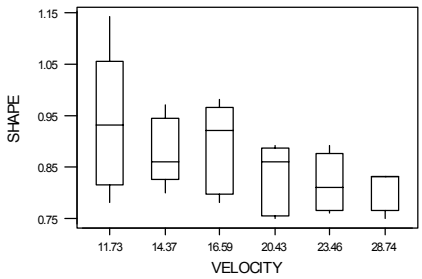


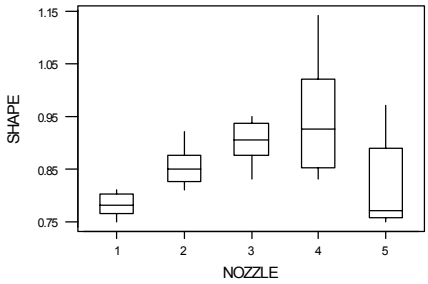
Section 13-4

13-25. a) Analysis of Variance for SHAPE

Source	DF	SS	MS	F	P
NOZZLE	4	0.102180	0.025545	8.92	0.000
VELOCITY	5	0.062867	0.012573	4.39	0.007
Error	20	0.057300	0.002865		
Total	29	0.222347			

Reject H_0, and conclude that nozzle type affects the mean shape measurement.





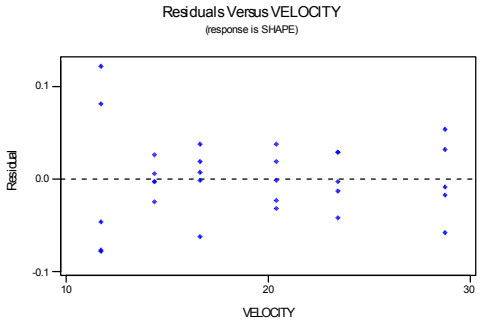
b) Fisher's pairwise comparisons

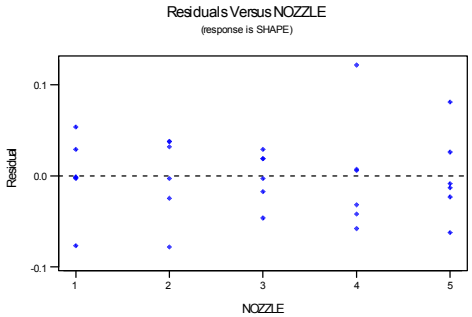
Family error rate = 0.268
Individual error rate = 0.0500
Critical value = 2.060
Intervals for (column level mean) - (row level mean)

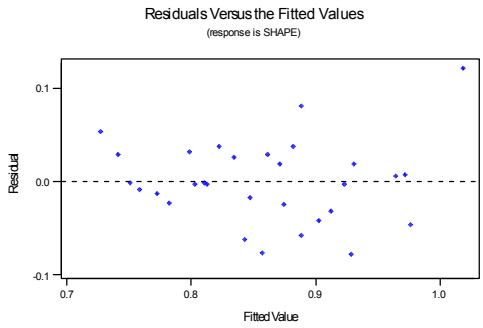
	1	2	3	4
2	-0.15412 0.01079			
3	-0.20246 -0.03754	-0.13079 0.03412		
4	-0.24412 -0.07921	-0.17246 -0.00754	-0.12412 0.04079	
5	-0.11412 0.05079	-0.04246 0.12246	0.00588 0.17079	0.04754 0.21246

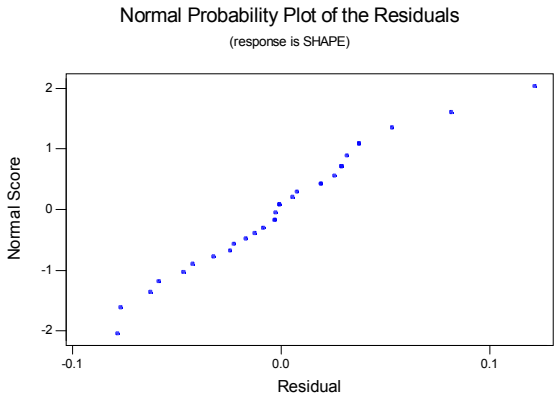
There are significant differences between nozzle types 1 and 3, 1 and 4, 2 and 4, 3 and 5, and 4 and 5.

c) The residual analysis shows that there is some inequality of variance. The normal probability plot is acceptable.









Supplemental Exercises

13-31. a)

```
Analysis of Variance for RESISTANCE
Source     DF        SS        MS        F        P
ALLOY       2   10941.8    5470.9    76.09    0.000
Error      27    1941.4      71.9
Total      29   12883.2
```

Reject H_0, the type of alloy has a significant effect on mean contact resistance.

b)

```
Fisher's pairwise comparisons
  Family error rate = 0.119
  Individual error rate = 0.0500
  Critical value = 2.052
  Intervals for (column level mean) - (row level mean)
                  1           2
     2      -13.58
         1.98
     3      -50.88      -45.08
            -35.32      -29.52
```

There are differences in the mean resistance for alloy types 1 and 3, and 2 and 3.

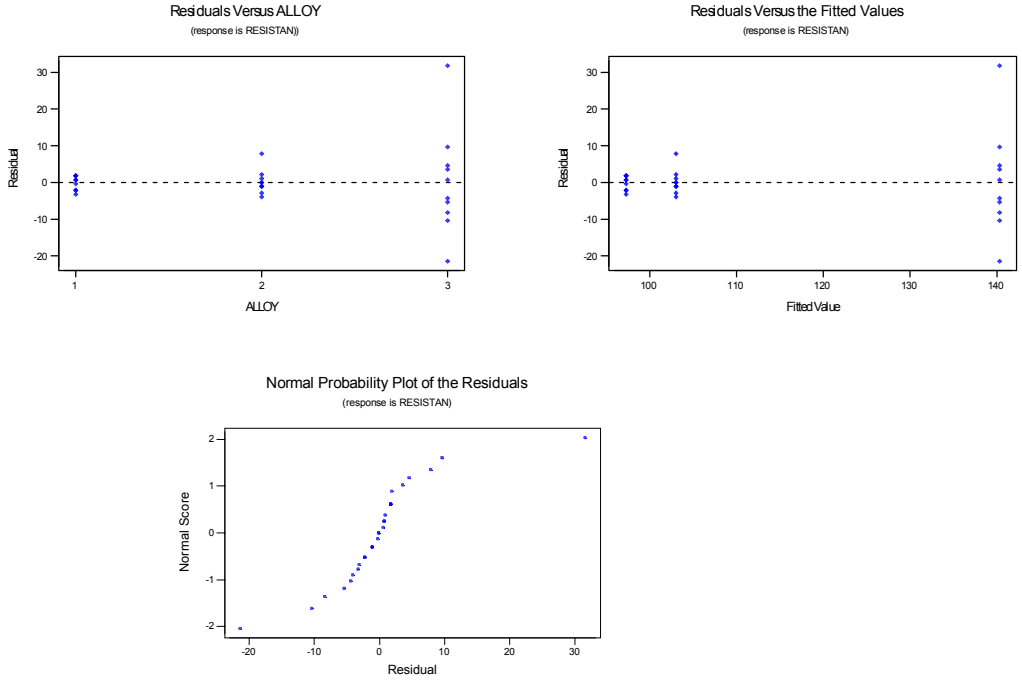
c) 99% confidence interval on the mean contact resistance for alloy 3

$$\bar{y}_3 - t_{0.005,271}\sqrt{\frac{MS_E}{n}} \le \mu_i \le \bar{y}_3 + t_{0.005,27}\sqrt{\frac{MS_E}{n}}$$

$$140.4 - 2.771\sqrt{\frac{71.9}{10}} \le \mu_3 \le 140.4 - 2.771\sqrt{\frac{71.9}{10}}$$

$$132.97 \le \mu_1 \le 147.83$$

d) Variability of the residuals increases with the response. The normal probability plot has some curvature in the tails, indicating a problem with the normality assumption. A transformation of the response should be conducted.



13-35. a)

```
Analysis of Variance for VOLUME
Source         DF        SS        MS        F        P
TEMPERATURE     2     16480      8240     7.84    0.007
Error          12     12610      1051
Total          14     29090
```

Reject H_0.

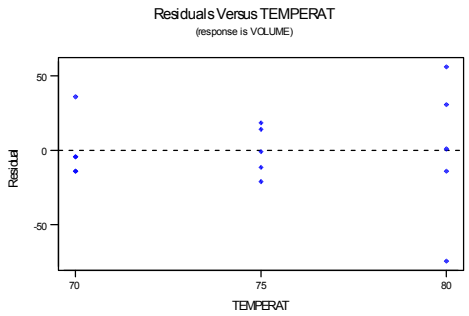
b) *P-value* = 0.007

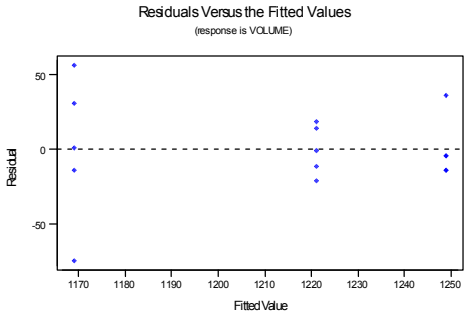
c)

```
Fisher's pairwise comparisons
  Family error rate = 0.116
  Individual error rate = 0.0500
  Critical value = 2.179
  Intervals for (column level mean) - (row level mean)
                  70          75
      75       -16.7
                72.7
      80        35.3         7.3
               124.7        96.7
```

There are significant differences in the mean volume for temperature levels 70 and 80, and 75 and 80. The highest temperature (80%) results in the smallest mean volume.

d)There are some relatively small differences in the variability at the different levels of temperature. The variability decreases with the fitted values. There is an unusual observation on the normal probability plot.



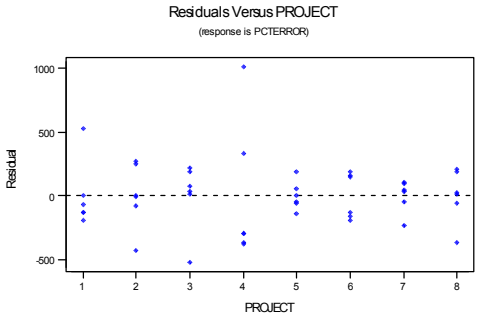


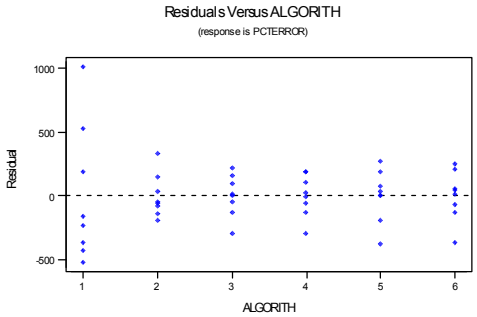
13-37. a) Analysis of Variance for PCTERROR

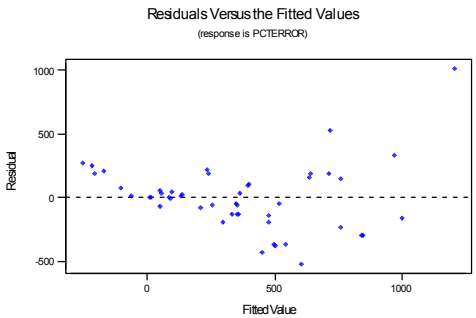
Source	DF	SS	MS	F	P
ALGORITH	5	2825746	565149	6.23	0.000
PROJECT	7	2710323	387189	4.27	0.002
Error	35	3175290	90723		
Total	47	8711358			

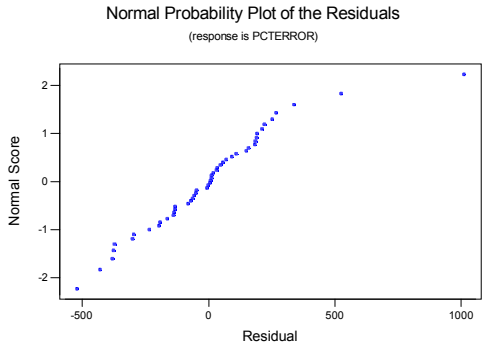
Reject H_0 , the algorithm is significant.

b) The residuals look acceptable, except there is one unusual point.









c) The best choice is algorithm 5 because it has the smallest mean and a low variablity.

13-39 a) $\lambda = \sqrt{1 + \frac{4(2\sigma^2)}{\sigma^2}} = 3$

From Chart VIII with numerator degrees of freedom = a - 1 = 4, denominator degrees of freedom = a(n - 1) = 15, $\beta = 0.15$, and the power = $1 - \beta = 0.85$.

b)

n	λ	a(n - 1)	β	Power = 1 - β
5	3.317	20	0.10	0.90

The sample size should be approximately n = 5