Chapter 14 Selected Problem Solutions

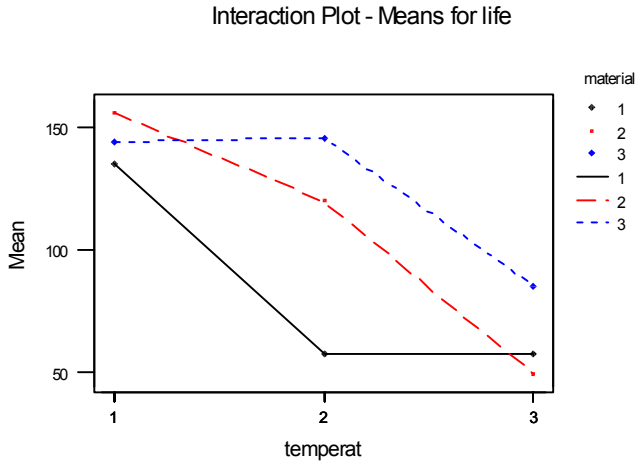
Section 14-3

14-1. a) Analysis of Variance for life

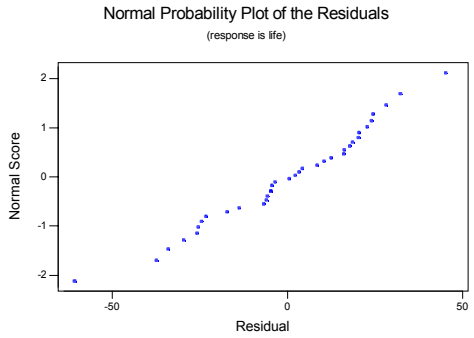
Source	DF	SS	MS	F	P
material	2	10683.7	5341.9	7.91	0.002
temperat	2	39118.7	19559.4	28.97	0.000
material*temperat	4	9613.8	2403.4	3.56	0.019
Error	27	18230.7	675.2		
Total	35	77647.0			

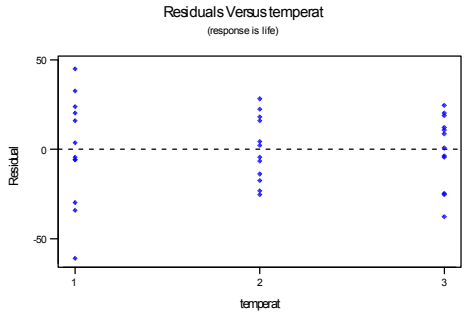
Main factors and interaction are all significant.

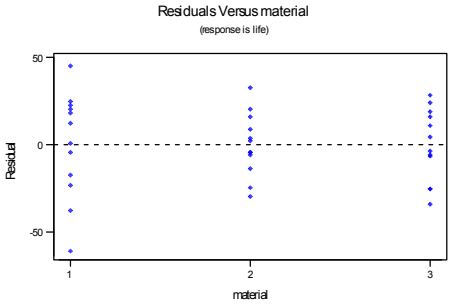
b)The mean life for material 2 is the highest at temperature level 1, in the middle at temperature level 2 and the lowest at temperature level 3. This shows that there is an interaction.



c) There appears to be slightly more variability at temperature 1 and material 1. The normal probability plot shows that the assumption of normality is reasonable.







14-3 a) $H_0 : \tau_1 = \tau_2 = 0$

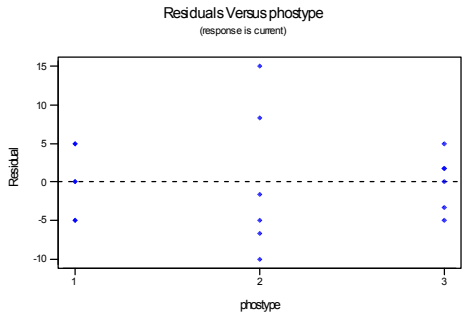
H_1 : at least one $\tau_i \neq 0$

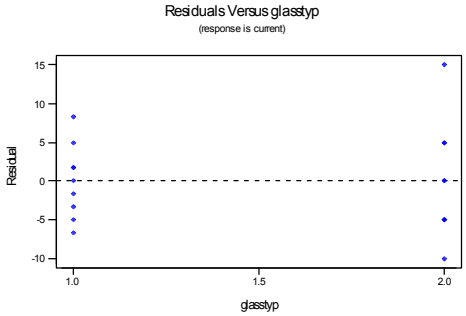
b) Analysis of Variance for current

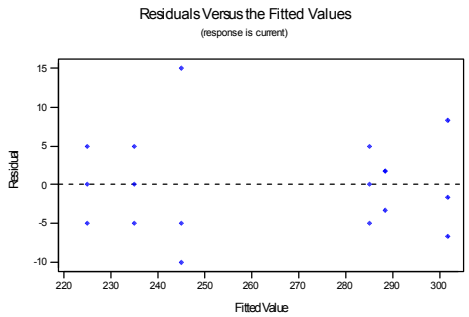
Source	DF	SS	MS	F	P
glasstyp	1	14450.0	14450.0	273.79	0.000
phostype	2	933.3	466.7	8.84	0.004
glasstyp*phostype	2	133.3	66.7	1.26	0.318
Error	12	633.3	52.8		
Total	17	16150.0			

Main effects are significant, the interaction is not significant. Glass type 1 and phosphor type 2 lead to the high mean current (brightness).

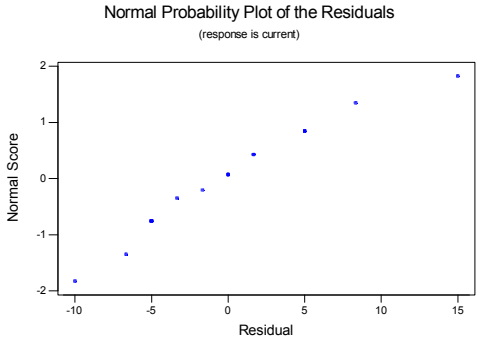
c) There appears to be slightly more variability at phosphor type 2 and glass type 2. The normal plot of the residuals shows that the assumption of normality is reasonable.







14-7 The ratio



$T = \dfrac{\bar{y}_{.i.} - \bar{y}_{.j.} - (\mu_i - \mu_j)}{\sqrt{2MS_E / n}}$ has a t distribution with $ab(n\text{-}1)$ degrees of freedom

Therefore, the (1-α)% confidence interval on the difference in two treatment means is

$$\bar{y}_{.i.} - \bar{y}_{.j.} - t_{a/2,ab(n-1)}\sqrt{\frac{2MS_E}{n}} \le \mu_i - \mu_j \le \bar{y}_{.i.} - \bar{y}_{.j.} + t_{a/2,ab(n-1)}\sqrt{\frac{2MS_E}{n}}$$

For exercise 14-6, the mean warping at 80% copper concentration is 21.0 and the mean warping at 60% copper concentration is 18.88 a=4, b=4, n=2 and MS_E=6.78. The degrees of freedom are (4)(4)(1)=16

$$(21.0 - 18.88) - 2.120\sqrt{\frac{2*6.78}{2}} \le \mu_3 - \mu_2 \le (21.0 - 18.88) + 2.120\sqrt{\frac{2*6.78}{2}}$$

$$-3.40 \le \mu_3 - \mu_2 \le 7.64$$

Therefore, there is no significant differences between the mean warping values at 80% and 60% copper concentration.

Section 14-4

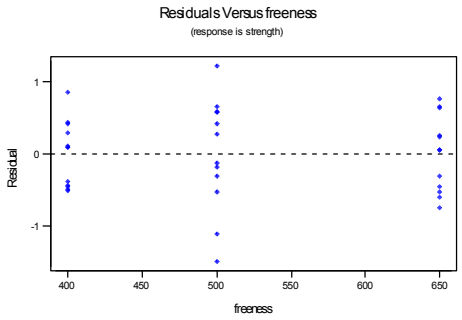
14-11 Parts a. and b.

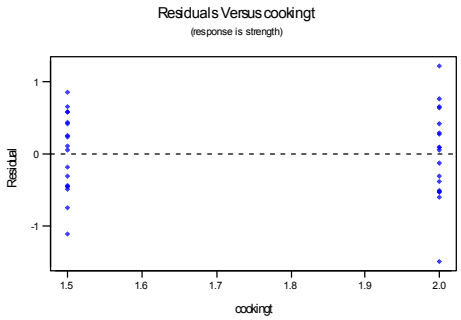
Analysis of Variance for strength

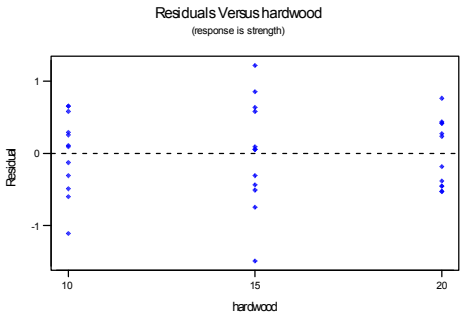
Source	DF	SS	MS	F	P
hardwood	2	8.3750	4.1875	7.64	0.003
cookingtime	1	17.3611	17.3611	31.66	0.000
freeness	2	21.8517	10.9258	19.92	0.000
hardwood*cookingtime	2	3.2039	1.6019	2.92	0.075
hardwood*freeness	4	6.5133	1.6283	2.97	0.042
cookingtime*freeness	2	1.0506	0.5253	0.96	0.399
Error	22	12.0644	0.5484		
Total	35	70.4200			

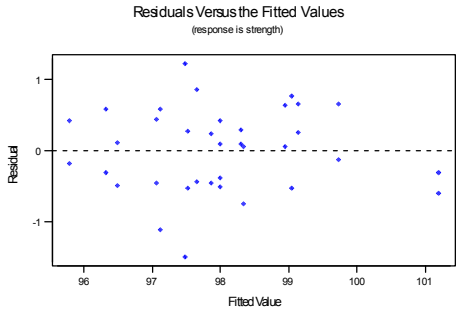
All main factors are significant. The interaction of hardwood * freeness is also significant.

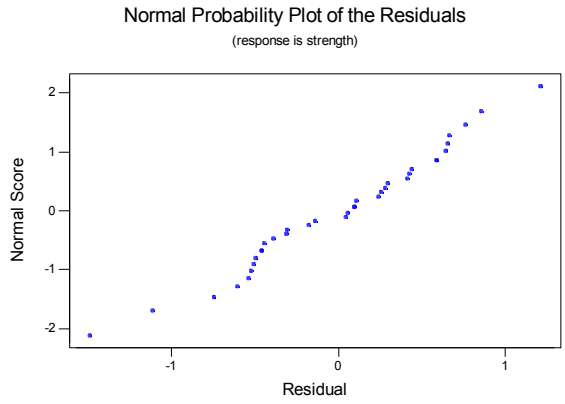
c) The residual plots show no serious problems with normality or equality of variance











Section 14-5

14-13 a) Analysis of Variance for life (coded units)

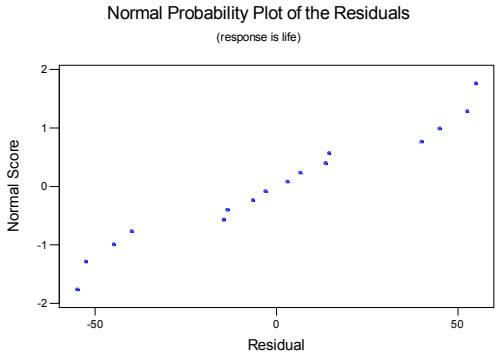
Source	DF	SS	MS	F	P
speed	1	1332	1332	0.49	0.502
hardness	1	28392	28392	10.42	0.010
angle	1	20592	20592	7.56	0.023
speed*hardness	1	506	506	0.19	0.677
speed*angle	1	56882	56882	20.87	0.000
hardness*angle	1	2352	2352	0.86	0.377
Error	9	24530	2726		
Total	15	134588			

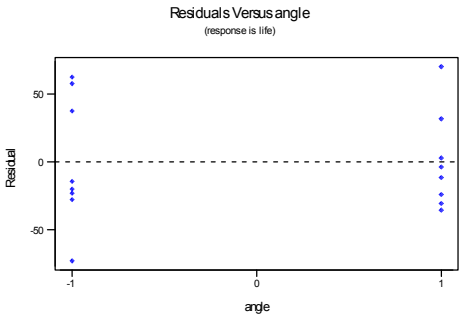
b) Estimated Effects and Coefficients for life (coded units)

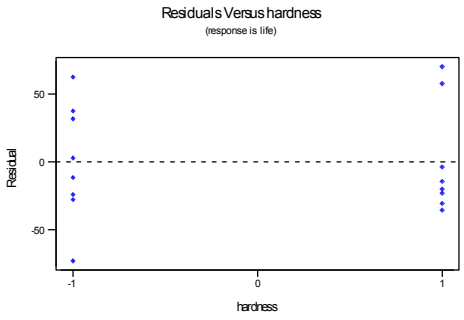
Term	Effect	Coef	SE Coef	T	P
Constant		413.13	12.41	33.30	0.000
speed	18.25	9.12	12.41	0.74	0.483
hardness	84.25	42.12	12.41	3.40	0.009
angle	71.75	35.87	12.41	2.89	0.020
speed*hardness	-11.25	-5.63	12.41	-0.45	0.662
speed*angle	-119.25	-59.62	12.41	-4.81	0.001
hardness*angle	-24.25	-12.12	12.41	-0.98	0.357
speed*hardness*angle	-34.75	-17.37	12.41	-1.40	0.199

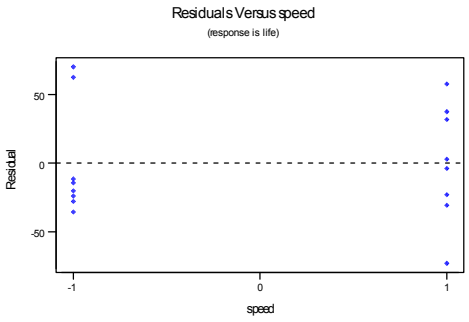
$$\hat{y} = 413.125 + 9.125x_1 + 45.12x_2 + 35.87x_3 - 59.62x_{13}$$

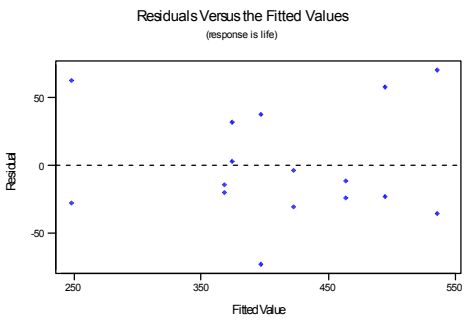
c) Analysis of the residuals shows that all assumptions are reasonable.



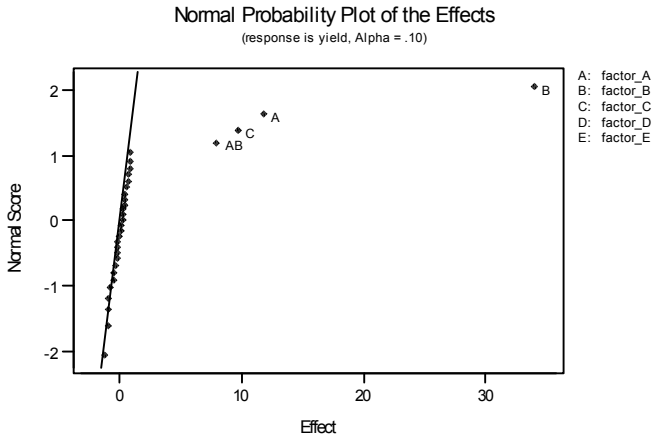








14-19. a) Factors A, B, C, and the interaction AB appear to be significant from the normal probability plot of the effects.



b)

Analysis of Variance for yield (coded units)

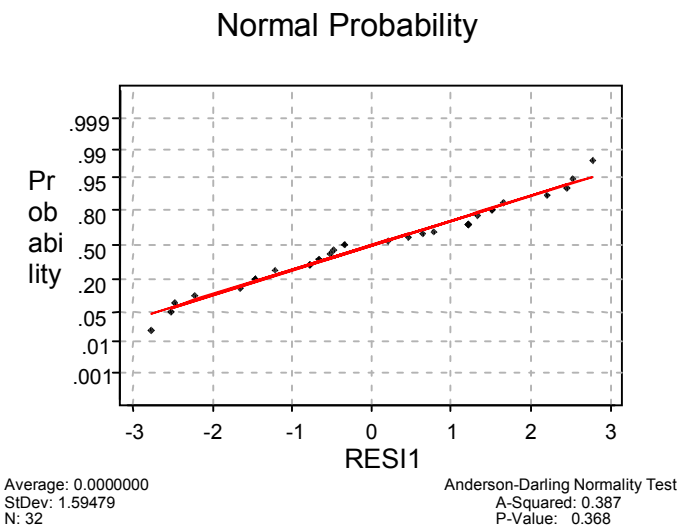
Term	Effect	Coef	StDev Coef	T	P
Constant		30.5312	0.2786	109.57	0.000
factor_A	11.8125	5.9063	0.2786	21.20	0.000
factor_B	9.6875	4.8437	0.2786	17.38	0.000
factor_D	-0.8125	-0.4063	0.2786	-1.46	0.164
factor_E	0.4375	0.2187	0.2786	0.79	0.444
factor_A*factor_B	7.9375	3.9687	0.2786	14.24	0.000
factor_A*factor_C	0.4375	0.2187	0.2786	0.79	0.444
factor_A*factor_D	-0.0625	-0.0313	0.2786	-0.11	0.912
factor_A*factor_E	0.9375	0.4687	0.2786	1.68	0.112
factor_B*factor_C	0.0625	0.0313	0.2786	0.11	0.912
factor_B*factor_D	-0.6875	-0.3437	0.2786	-1.23	0.235
factor_B*factor_E	0.5625	0.2813	0.2786	1.01	0.328
factor_C*factor_D	0.8125	0.4062	0.2786	1.46	0.164
factor_C*factor_E	0.3125	0.1563	0.2786	0.56	0.583
factor_D*factor_E	-1.1875	-0.5938	0.2786	-2.13	0.049

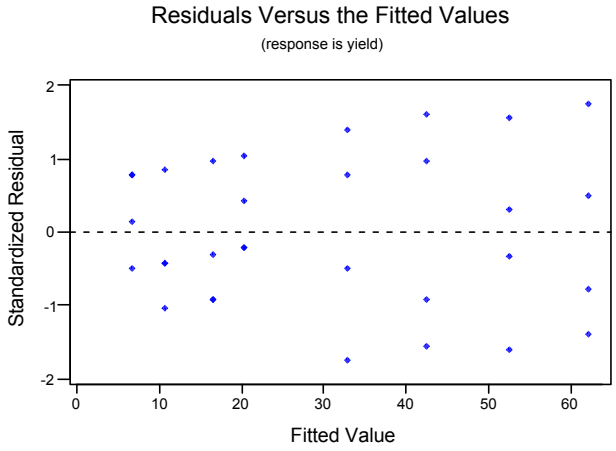
Analysis of Variance for yield

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	5	11087.9	11087.9	2217.58	892.61	0.000
2-Way Interactions	10	536.3	536.3	53.63	21.59	0.000
Residual Error	16	39.7	39.7	2.48		
Total	31	11664.0				

The analysis confirms our findings from part a)

c) The normal probability plot of the residuals is satisfactory. However their variance appears to increase as the fitted value increases.





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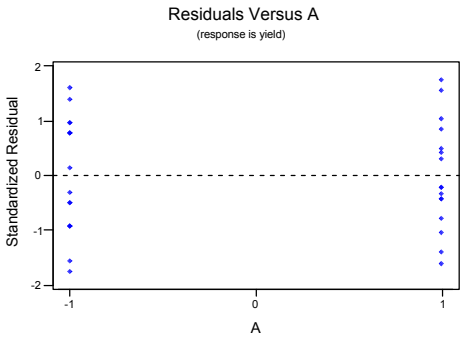
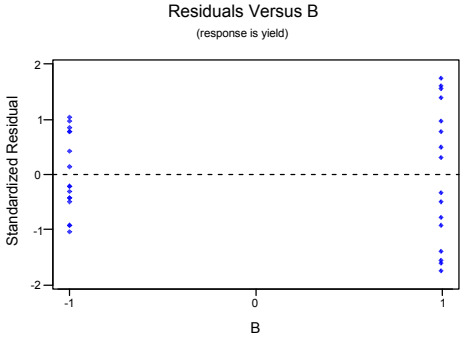
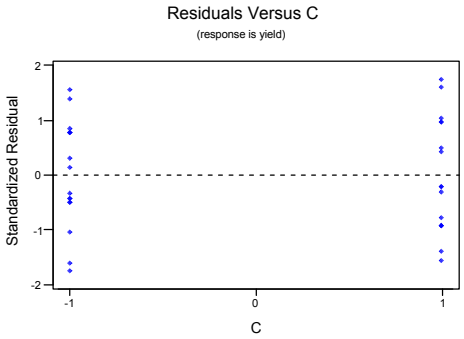
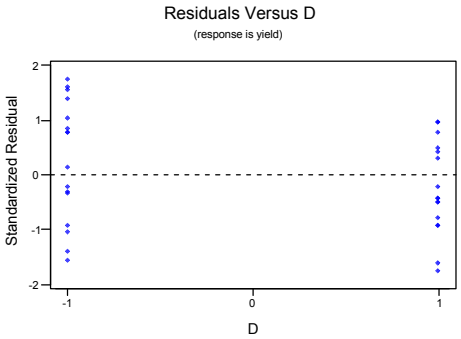
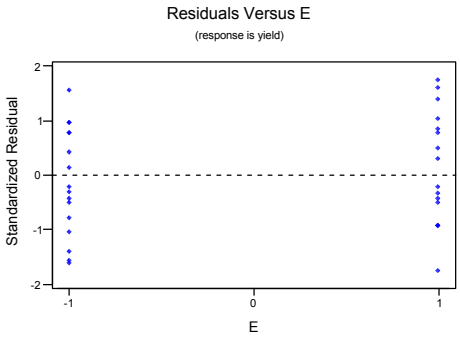
d) All plots support the constant variance assumption , although there is a very slight indication that variability is greater at the high level of factor B.



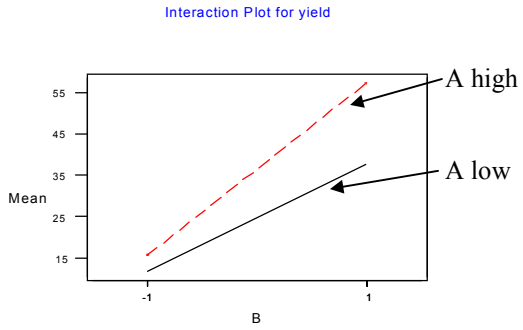






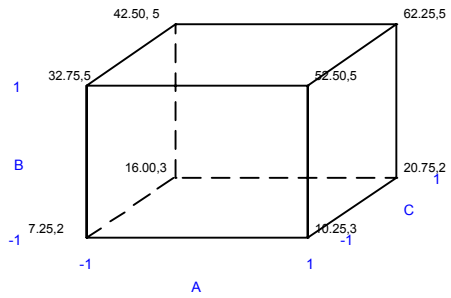


e) The AB interaction appears to be significant. The interaction plot from MINITAB indicates that a high level of A and of B increases the mean yield, while low levels of both factors would lead to a reduction in the mean yield.

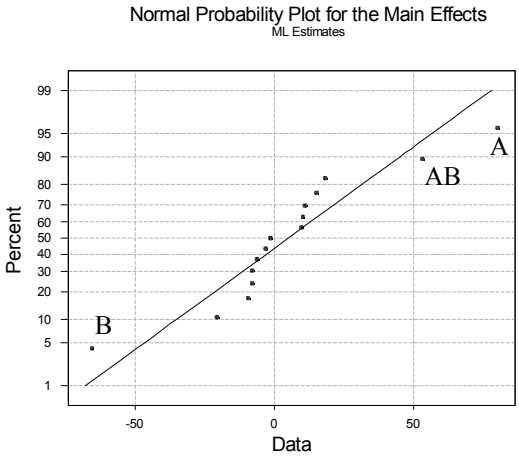


f.) To increase yield and therefor optimize the process, we would want to set A, B, and C at their high levels.

g) It is evident from the cube plot that we should run the process with all factors set at their high level.



14-21



b) Based on the normal probability plot of the effects, factors A, B and AB are significant. The model would include these three factors.

c) The estimated model is: $\hat{y} = 400 + 40.124x_1 - 32.75x_2 + 26.625x_{12}$

Section 14-6

14-25 Model with four blocks

BLOCK	A	B	C	D	var_1
1	-1	-1	-1	-1	190
1	1	-1	1	-1	181
1	-1	1	-1	1	187
1	1	1	1	1	180
2	1	-1	-1	-1	174
2	-1	-1	1	-1	177
2	1	1	-1	1	185
2	-1	1	1	1	187
3	-1	1	-1	-1	181
3	1	1	1	-1	173
3	-1	-1	-1	1	198
3	1	-1	1	1	179
4	1	1	-1	-1	183
4	-1	1	1	-1	188
4	1	-1	-1	1	172
4	-1	-1	1	1	199

```
Term                           Effect      Coef
Constant                                183.375
Block                                    -1.625
factor_A                      -10.000    -5.000
factor_B                       -0.750    -0.375
factor_C                       -0.750    -0.375
factor_D                        5.000     2.500
factor_A*factor_B               4.500     2.250
factor_A*factor_C               0.500     0.250
factor_A*factor_D              -3.750    -1.875
factor_B*factor_C              -1.250    -0.625
factor_B*factor_D              -1.500    -0.750
factor_C*factor_D               1.500     0.750
factor_A*factor_B*factor_C     -6.000    -3.000
factor_A*factor_B*factor_D      4.750     2.375
factor_A*factor_C*factor_D     -0.250    -0.125
factor_B*factor_C*factor_D     -2.000    -1.000

Term         Effect      Coef  StDev Coef       T      P
Constant             183.375       1.607  114.14  0.000
Block                 -1.625       1.607   -1.01  0.336
factor_A    -10.000    -5.000       1.607   -3.11  0.011
factor_B     -0.750    -0.375       1.607   -0.23  0.820
factor_C     -0.750    -0.375       1.607   -0.23  0.820
factor_D      5.000     2.500       1.607    1.56  0.151

Analysis of Variance for var_1
Source               DF      Seq SS     Adj SS     Adj MS      F      P
Blocks                1       42.25      42.25      42.25   1.02  0.336
Main Effects          4      504.50     504.50     126.13   3.05  0.069
Residual Error       10      413.00     413.00      41.30
Total                15      959.75
```

Factor A is the only significant factor.

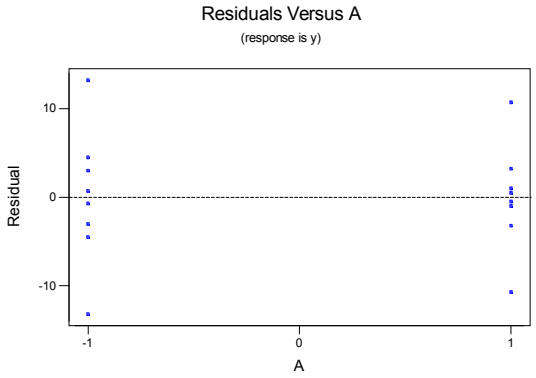
14-29 a)

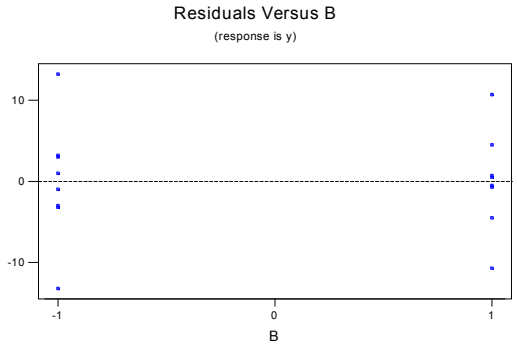
```
Estimated Effects and Coefficients for y
Term         Effect      Coef  StDev Coef       T      P
Constant                56.37       2.633   21.41  0.000
Block  1                15.63       4.560    3.43  0.014
       2                -3.38       4.560   -0.74  0.487
```

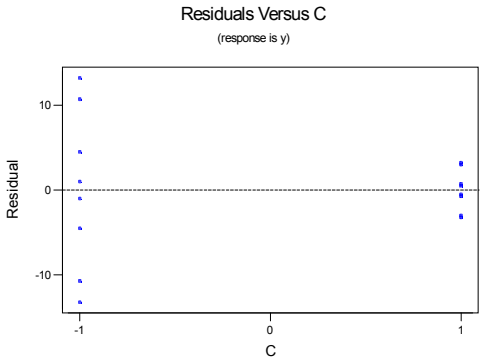
```
       3              -10.88       4.560   -2.38  0.054
A           -45.25    -22.62       2.633   -8.59  0.000
B            -1.50     -0.75       2.633   -0.28  0.785
C            14.50      7.25       2.633    2.75  0.033
A*B          19.00      9.50       2.633    3.61  0.011
A*C         -14.50     -7.25       2.633   -2.75  0.033
B*C          -9.25     -4.63       2.633   -1.76  0.130

Analysis of Variance for y
Source                DF      Seq SS     Adj SS     Adj MS      F      P
Blocks                 3      1502.8     1502.8      500.9   4.52  0.055
Main Effects           3      9040.2     9040.2     3013.4  27.17  0.001
2-Way Interactions     3      2627.2     2627.2      875.7   7.90  0.017
Residual Error         6       665.5      665.5      110.9
Total                 15     13835.7
```

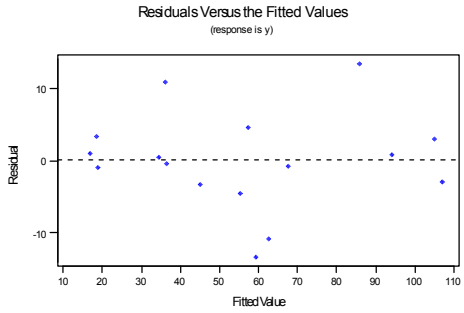
Factors A, C, AB, and AC are significant.

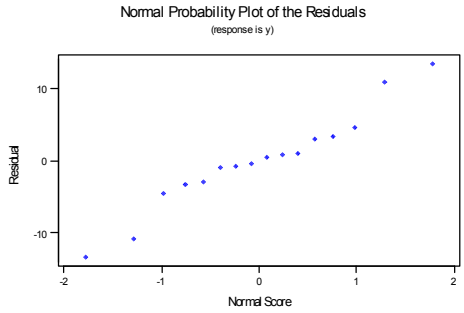
b) Analysis of the residuals shows that the model is adequate. There is more variability on the response associated with the low setting of factor C, but that is the only problem.









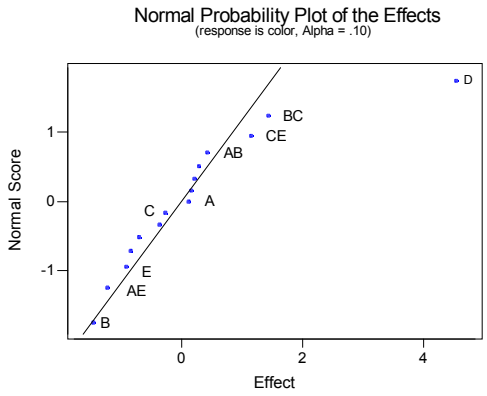


c.) Some of the information from the experiment is lost because the design is run in 4 blocks. This causes us to lose information on the ABC interaction even though we have replicated the experiment twice. If it is possible to run the experiment in only 2 blocks, there would be information on all interactions.

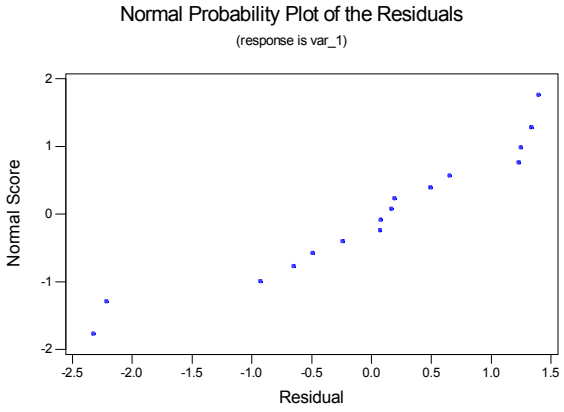
d) To have data on all interactions, we could run the experiment so that each replicate is a block (therefore only 2 blocks).

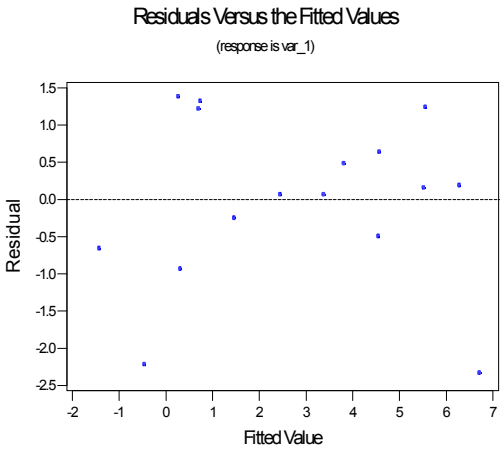
Section 14-7

14-31 a) Factors A, B and D are active factors.



b) There are no serious problems with the residual plots. The normal probability plot has a little bit of curvature at the low end and there is a little more variability at the lower and higher ends of the fitted values.



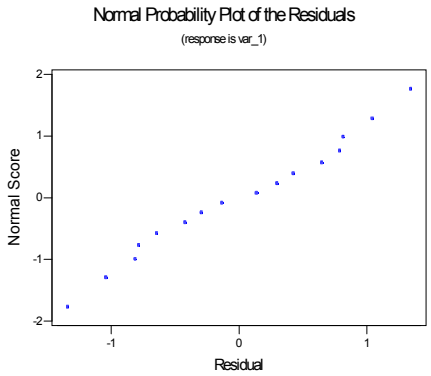


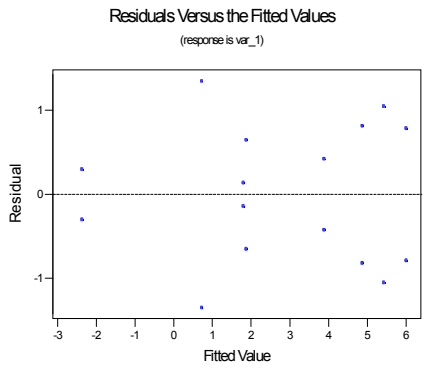
c) Part a. indicates that only A,B, and D are important. In these factors only, the design is a 2^3 with two replicates.

```
Estimated Effects and Coefficients for var_1
Term                          Effect      Coef  StDev Coef       T      P
Constant                              2.7700      0.2762   10.03  0.000
factor_A                      1.4350  0.7175      0.2762    2.60  0.032
```

```
factor_B                      -1.4650   -0.7325    0.2762   -2.65  0.029
factor_D                       4.5450    2.2725    0.2762    8.23  0.000
factor_A*factor_B              1.1500    0.5750    0.2762    2.08  0.071
factor_A*factor_D             -1.2300   -0.6150    0.2762   -2.23  0.057
factor_B*factor_D              0.1200    0.0600    0.2762    0.22  0.833
factor_A*factor_B*factor_D    -0.3650   -0.1825    0.2762   -0.66  0.527

Analysis of Variance for var_1
Source                DF      Seq SS     Adj SS     Adj MS      F      P
Main Effects           3      99.450    99.4499    33.1500  27.15  0.000
2-Way Interactions     3      11.399    11.3992     3.7997   3.11  0.088
3-Way Interactions     1       0.533     0.5329     0.5329   0.44  0.527
Residual Error         8       9.767     9.7668     1.2208
  Pure Error           8       9.767     9.7668     1.2208
Total                 15     121.149
```

Factors A, B, D, AB and AD are significant.



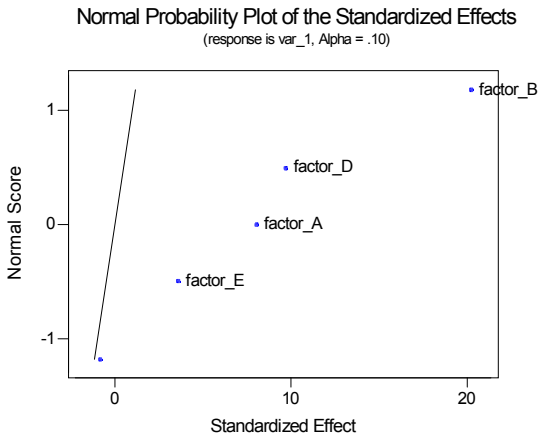


The normal probability plot and the plot of the residuals versus fitted values are satisfactory.

14-35 Since factors A, B, C, and E form a word in the complete defining relation, it can be verified that the resulting design is two replicates of a 2^{4-1} fractional factorial. This is different than the design that results when C and E are dropped from the 2^{6-2} in Table 14-28 which results in a full factorial because, the factors ABDF do not form a word in the complete defining relation

14-37 Generators D=AB, E=AC for 2^{5-2}, Resolution III

A	B	C	D	E	var_1
-1	-1	-1	1	1	1900
1	-1	-1	-1	-1	900
-1	1	-1	-1	1	3500
1	1	-1	1	-1	6100
-1	-1	1	1	-1	800
1	-1	1	-1	1	1200
-1	1	1	-1	-1	3000
1	1	1	1	1	6800



Estimated Effects and Coefficients for var_1 (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		3025.00	90.14	33.56	0.001
factor_A	1450.00	725.00	90.14	8.04	0.015
factor_B	3650.00	1825.00	90.14	20.25	0.002
factor_C	-150.00	-75.00	90.14	-0.83	0.493
factor_D	1750.00	875.00	90.14	9.71	0.010
factor_E	650.00	325.00	90.14	3.61	0.069

Analysis of Variance for var_1 (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	5	37865000	37865000	7573000	116.51	0.009
Residual Error	2	130000	130000	65000		
Total	7	37995000				

Factors A, B and D are significant.

Supplemental Exercises

14-41 a Estimated Effects and Coefficients for var_1 (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		191.563	1.158	165.49	0.000
factor_A (PH)	5.875	2.937	1.158	2.54	0.026
factor_B (CC)	-0.125	-0.062	1.158	-0.05	0.958
factor_A*factor_B	11.625	5.812	1.158	5.02	0.000

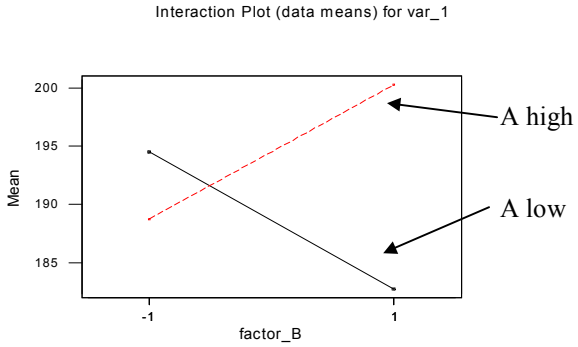
Analysis of Variance for var_1 (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	2	138.125	138.125	69.06	3.22	0.076
2-Way Interactions	1	540.562	540.562	540.56	25.22	0.000
Residual Error	12	257.250	257.250	21.44		
Pure Error	12	257.250	257.250	21.44		
Total	15	935.938				

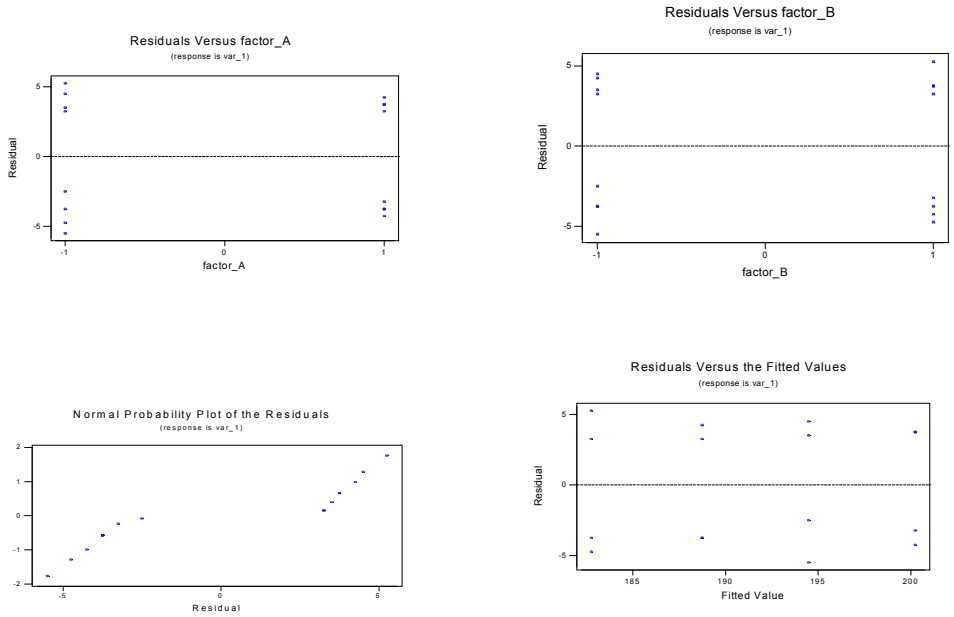
The main effect of pH and the interaction of pH and Catalyst Concentration (CC) are significant at the 0.05 level of significance.

The model used is viscosity = $191.563 + 2.937x_1 - 0.062x_2 + 5.812x_{12}$

b.) The interaction plot shows that there is a strong interaction. When Factor A is at its low level, the mean response is large at the low level of B and is small at the high level of B. However, when A is at its high level, the results are opposite.



c.) The plots of the residuals show that the equality of variance assumption is reasonable. However, there is a large gap in the middle of the normal probability plot. Sometimes, this can indicate that there is another variable that has an effect on the response but which is not included in the experiment. For example, in this experiment, note that the replicates in each cell have two pairs of values that are very similar, but there is a rather large difference in the mean values of the two pairs. (Cell 1 has 189 and 192 as one pair and 198 and 199 as the other.)

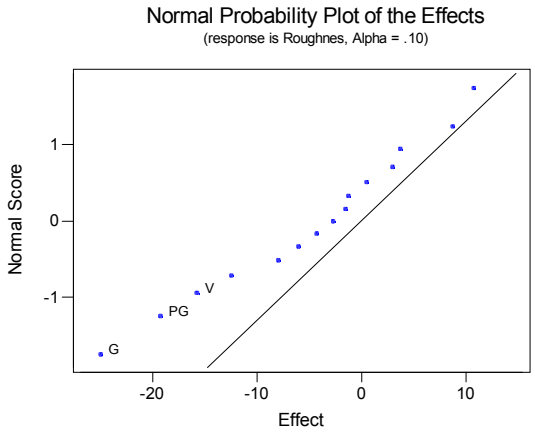


14-47 a)

Term	Effect
V	-15.75

F	8.75
P	10.75
G	-25.00
V*F	3.00
V*P	-8.00
V*G	-2.75
F*P	-6.00
F*G	3.75
P*G	-19.25
V*F*P	-1.25
V*F*G	0.50
V*P*G	-1.50
F*P*G	-12.50
V*F*P*G	-4.25

b)



According to the probability plot, factors V, P, and G and, PG are possibly significant.

Estimated Effects and Coefficients for roughnes (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		102.75	2.986	34.41	0.000
V	-15.75	-7.87	2.986	-2.64	0.046
F	8.75	4.37	2.986	1.46	0.203
P	10.75	5.37	2.986	1.80	0.132
G	-25.00	-12.50	2.986	-4.19	0.009
V*F	3.00	1.50	2.986	0.50	0.637
V*P	-8.00	-4.00	2.986	-1.34	0.238
V*G	-2.75	-1.38	2.986	-0.46	0.665
F*P	-6.00	-3.00	2.986	-1.00	0.361
F*G	3.75	1.88	2.986	0.63	0.558
P*G	-19.25	-9.63	2.986	-3.22	0.023

Analysis of Variance for roughnes (coded units)

Analysis of Variance for Roughnes (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	4	4260.7	4260.7	1065.2	7.46	0.024

```
2-Way Interactions      6       2004.7      2004.7       334.1    2.34   0.184
Residual Error          5        713.5       713.5       142.7
Total                  15       6979.0
```

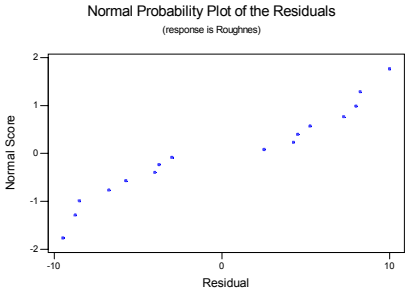
$$\hat{y} = 102.75 - 7.87x_1 + 5.37x_3 - 12.50x_4 - 9.63x_{34}$$

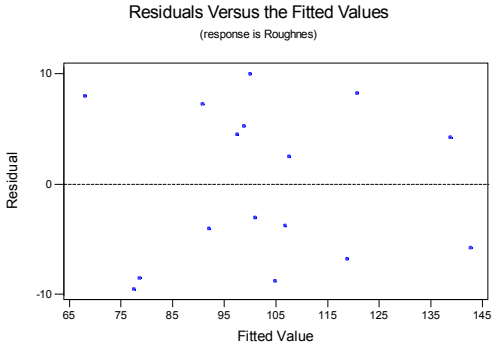
c) From the analysis, we see that water jet pressure (P), abrasive grain size (G), and jet traverse speed (V) are significant along with the interaction of water jet pressure and abrasive grain size

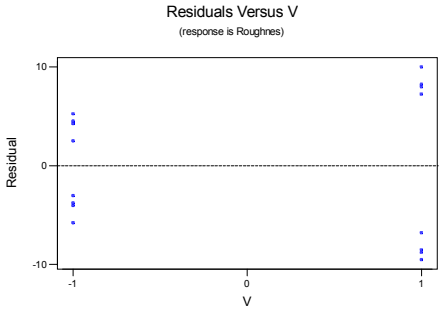
d) The residual plots appear to indicate the assumption of constant variance may not be met. The assumption of normality appears reasonable.

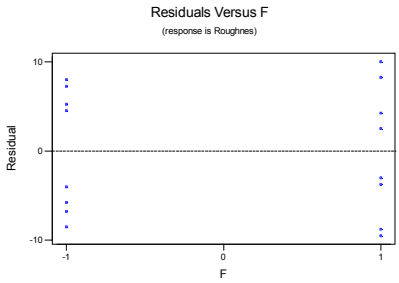
14-49 The design uses G=VPF as the generator.

```
Alias Structure
```

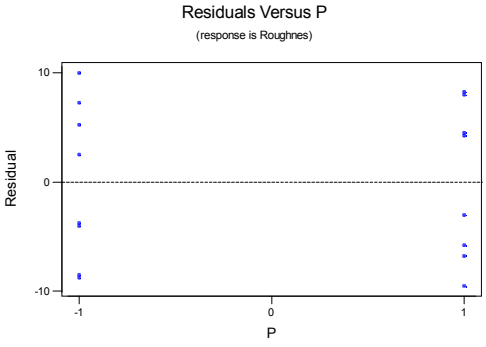


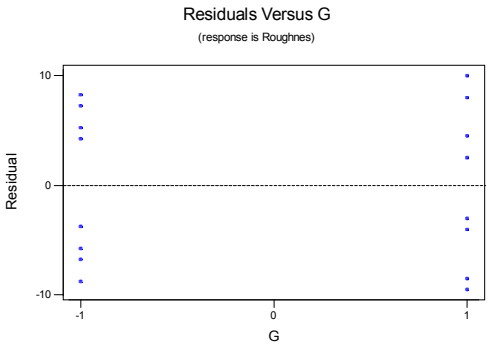







```
I + V*P*F*G
```






```
V + P*F*G
P + V*F*G
F + V*P*G
G + V*P*F
```

```
V*P + F*G
V*F + P*G
V*G + P*F


Estimated Effects and Coefficients for C9 (coded units)

Term          Effect      Coef     SE Coef       T      P
Constant                102.63       6.365   16.12  0.004
V            -14.75     -7.37       6.365   -1.16  0.366
P            -28.25    -14.12       6.365   -2.22  0.157
F             -1.25     -0.62       6.365   -0.10  0.931
G            -14.75     -7.38       6.365   -1.16  0.366
P*G           17.75      8.88       6.365    1.39  0.298

Analysis of Variance for C9 (coded units)

Source                DF      Seq SS     Adj SS     Adj MS      F      P
Main Effects           4      2469.5     2469.5      617.4   1.90  0.373
2-Way Interactions     1       630.1      630.1      630.1   1.94  0.298
Residual Error         2       648.3      648.3      324.1
Total                  7      3747.9
```

The results do not show any significant factors. A lot of the information is lost due to the half-fraction of the design.

14-51

Design Generators: D = AB E = AC

Alias Structure

I + ABD + ACE + BCDE

A + BD + CE + ABCDE
B + AD + CDE + ABCE
C + AE + BDE + ABCD
D + AB + BCE + ACDE
E + AC + BCD + ABDE
BC + DE + ABE + ACD
BE + CD + ABC + ADE

Design

StdOrder	A	B	C	D	E
1	-1	-1	-1	1	1
2	1	-1	-1	-1	-1
3	-1	1	-1	-1	1
4	1	1	-1	1	-1
5	-1	-1	1	1	-1
6	1	-1	1	-1	1
7	-1	1	1	-1	-1
8	1	1	1	1	1