

Chapter 15 Selected Problem Solutions

Section 15-2

- 15-1. 1. The parameter of interest is median of pH.
 2. $H_0 : \tilde{\mu} = 7.0$
 3. $H_1 : \tilde{\mu} \neq 7.0$
 4. $\alpha=0.05$
 5. The test statistic is the observed number of plus differences or $r^+ = 8$.
 6. We reject H_0 if the *P-value* corresponding to $r^+ = 8$ is less than or equal to $\alpha=0.05$.
 7. Using the binomial distribution with $n=10$ and $p=0.5$, $P\text{-value} = 2P(R^* \geq 8 | p=0.5) = 0.109$
 8. Conclusion: we cannot reject H_0 . There is not enough evidence to reject the manufacturer's claim that the median of the pH is 7.0

- 15-5 a. 1. The parameter of interest is the median compressive strength
 2. $H_0 : \tilde{\mu} = 2250$
 3. $H_1 : \tilde{\mu} > 2250$
 4. $\alpha=0.05$
 5. The test statistic is the observed number of plus differences or $r^+ = 7$.
 6. We reject H_0 if the *P-value* corresponding to $r^+ = 7$ is less than or equal to $\alpha=0.05$.
 7. Using the binomial distribution with $n=12$ and $p=0.5$, $P\text{-value} = P(R^* \geq 7 | p=0.5) = .3872$
 8. Conclusion: cannot reject H_0 . The median compressive strength is not more than 2250.

- b. 1. The parameter of interest is the median compressive strength

2. $H_0 : \tilde{\mu} = 2250$

3. $H_1 : \tilde{\mu} > 2250$

4. $\alpha=0.05$

5. Test statistic is $z_0 = \frac{r^+ - 0.5n}{0.5\sqrt{n}}$

6. We reject H_0 if the $|Z_0| > Z_{0.025} = 1.96$

7. Computation: $z_0 = \frac{7 - 0.5(12)}{0.5\sqrt{12}} = 0.577$

8. Conclusion: cannot reject H_0 . The median compressive strength is not more than 2250.

The *P-value* = $1 - \Phi(0.58) = 1 - .7190 = 0.281$

- 15-7. 1. The parameter of interest is the median titanium content

2. $H_0 : \tilde{\mu} = 8.5$

3. $H_1 : \tilde{\mu} \neq 8.5$

4. $\alpha=0.05$

5. Test statistic is $z_0 = \frac{r^+ - 0.5n}{0.5\sqrt{n}}$

6. We reject H_0 if the $|Z_0| > Z_{0.025} = 1.96$

7. Computation: $z_0 = \frac{7 - 0.5(20)}{0.5\sqrt{20}} = -1.34$

8. Conclusion: cannot reject H_0 . The median titanium content is 8.5.

The *P-value* = $2 * P(|Z| > 1.34) = 0.1802$.

- 15-9. 1. The parameters of interest are the median hardness readings for the two tips

2. $H_0 : \tilde{\mu}_D = 0$

3. $H_1 : \tilde{\mu}_D \neq 0$

4. $\alpha=0.05$

5. The test statistic is $r = \min(r^+, r^-)$.
 6. Since $\alpha=0.05$ and $n=8$, Appendix, Table VII gives the critical value of $r_{0.05}^* = 2$. We will reject H_0 in favor of H_1 if $r \leq 1$.
 7. $r^+ = 6$ and $r^- = 2$ and so $r = \min(6, 2) = 2$
 8. Conclusion: cannot reject H_0 . There is not significant difference in the tips.
- 15-11.
1. The parameters of interest are the median drying times for the two formulations.
 2. $H_0 : \tilde{\mu}_D = 0$
 3. $H_1 : \tilde{\mu}_D \neq 0$
 4. $\alpha=0.05$
 5. Test statistic is $z_0 = \frac{r^+ - 0.5n}{0.5\sqrt{n}}$
 6. We reject H_0 if the $|Z_0| > Z_{0.025} = 1.96$
 7. Computation: $z_0 = \frac{15 - 0.5(20)}{0.5\sqrt{20}} = 2.24$
 8. Conclusion: reject H_0 . There is a difference in the median drying times between the two formulations.
- The P -value = $2 * P(|Z| > 2.24) = 0.025$.
- 15-17.
- a) $\alpha = P(Z > 1.96) = 0.025$
 - b) $\beta = P\left(\frac{X}{\sigma / \sqrt{n}} = 1.96 \mid \mu = 1\right) = P(Z < -1.20) = 0.115$
 - c) The sign test that rejects if $R^- \leq 1$ has $\alpha = 0.011$ based on the binomial distribution.
 - d) $\beta = P(R^- > 1 \mid \mu = 1) = 0.1587$. Therefore, R^- has a binomial distribution with $p=0.1587$ and $n = 10$ when $\mu = 1$. Then $\beta = 0.487$. The value of β is greater for the sign test than for the normal test because the Z -test was designed for the normal distribution.

Section 15-3

- 15-21
1. The parameter of interest is the mean titanium content
 2. $H_0 : \mu = 8.5$
 3. $H_1 : \mu \neq 8.5$
 4. $\alpha=0.05$
 5. The test statistic is $w = \min(w^+, w^-)$.
 6. We will reject H_0 if $w \leq W_{0.05}^* = 52$, since $\alpha=0.05$ and $n=20$, the value in Appendix A, Table VIII.
 7. $w^+ = 71$ and $w^- = 102$ and so $w = \min(71, 102) = 71$
 8. Conclusion: Since $71 > 52$, we cannot reject H_0 .
- 15-23
1. The parameter of interest is the mean titanium content
 2. $H_0 : \mu = 2.5$
 3. $H_1 : \mu < 2.5$
 4. $\alpha=0.05$
 5. The test statistic $w = \min(w^+, w^-)$.
 6. We will reject H_0 if $w \leq W_{0.05}^* = 65$, since $\alpha=0.05$ and $n=22$ the value in Appendix A, Table VIII.
 7. $w^+ = 225$ and $w^- = 8$ and so $w = \min(225, 8) = 8$
 8. Conclusion: Since $8 < 65$, we reject H_0 .
- 15-27.
1. The parameters of interest are the mean blood cholesterol levels.
 2. $H_0 : \mu_D = 0$
 3. $H_1 : \mu_D \neq 0$
 4. $\alpha=0.05$
 5. The test statistic is $w = \min(w^+, w^-)$.

6. We will reject H_0 if $w \leq w_{0.05}^* = 25$, since $\alpha=0.05$ and $n=15$, the value in Appendix A, Table VIII.

7. $w^+ = 118$ and $w^- = 1$ and so $w = \min(118, 1) = 1$ Since $1 < 25$

8. Conclusion: Since $1 < 25$, we reject H_0 .

Section 15-4

15-31. 1. The parameters of interest are the mean image brightness'.

2. $H_0 : \mu_1 = \mu_2$

3. $H_1 : \mu_1 > \mu_2$

4. $\alpha=0.025$

5. The test statistic is $z_0 = \frac{W_1 - \mu_{w_1}}{\sigma_{w_1}}$

6. We will reject H_0 if $Z_0 > Z_{0.025} = 1.96$

7. $w_1 = 40$, $\mu_{w_1} = 85.5$ and $\sigma_{w_1}^2 = 128.25$

$$z_0 = \frac{54 - 85.5}{11.32} = -2.78$$

Since $Z_0 < 1.96$, cannot reject H_0

8. Conclusion: do not reject H_0 .

P-value = 0.9973

15-35. 1. The parameters of interest are the mean etch rates

2. $H_0 : \mu_1 = \mu_2$

3. $H_1 : \mu_1 \neq \mu_2$

4. $\alpha=0.025$

5. The test statistic is $z_0 = \frac{W_1 - \mu_{w_1}}{\sigma_{w_1}}$

6. We will reject H_0 if $|Z_0| > Z_{0.025} = 1.96$

7. $w_1 = 55$, $\mu_{w_1} = 105$ and $\sigma_{w_1}^2 = 175$

$$z_0 = \frac{55 - 105}{13.23} = -3.77$$

Since $|Z_0| > 1.96$, reject H_0

8. Conclusion: reject H_0 .

P-value = 0.0001

Section 15-5

15-37. Kruskal-Wallis Test on strength

mixingte	N	Median	Ave Rank	Z
1	4	2945	9.6	0.55
2	4	3075	12.9	2.12
3	4	2942	9.0	0.24
4	4	2650	2.5	-2.91
Overall	16		8.5	

H = 10.00 DF = 3 P = 0.019

H = 10.03 DF = 3 P = 0.018 (adjusted for ties)

* NOTE * One or more small samples

Reject H_0

Supplemental

- 15-43. 1. The parameter of interest is median of surface finish.
 2. $H_0 : \tilde{\mu} = 10.0$
 3. $H_1 : \tilde{\mu} \neq 10.0$
 4. $\alpha=0.05$
 5. The test statistic is the observed number of plus differences or $r^+ = 5$.
 6. We reject H_0 if the *P-value* corresponding to $r^+ = 5$ is less than or equal to $\alpha=0.05$.
 7. Using the binomial distribution with $n=10$ and $p=0.5$, $P\text{-value} = 2P(R^* \geq 5 | p=0.5) = 1.0$
 8. Conclusion: we cannot reject H_0 . We cannot reject the claim that the median is 10 μin .

- 15-45. The parameter of interest is the median fluoride emissions

$$H_0 : \tilde{\mu} = 6$$

$$H_1 : \tilde{\mu} < 6$$

$$\alpha = 0.05$$

Using Minitab (Sign Rank Test)

	N	Below	Equal	Above	P	Median
Y	15	9	2	4	0.1334	4.000

Do not reject H_0

- 15-47. 1. The parameters of interest are the median impurity levels.

$$2. H_0 : \tilde{\mu}_D = 0$$

$$3. H_1 : \tilde{\mu}_D \neq 0$$

$$4. \alpha = 0.01$$

5. The test statistic is $r = \min(r^+, r^-)$.

6. Since $\alpha=0.01$ and $n=8$, Appendix, Table VII gives the critical value of $r_{0.01}^* = 0$. We will reject H_0 in favor of H_1 if $r \leq 0$.

7. $r^+ = 1$ and $r^- = 7$ and so $r = \min(1, 7) = 1$

8. Conclusion: cannot reject H_0 . There is no significant difference in the impurity levels.

- 15-49. The parameter of interest is the median fluoride emissions

$$H_0 : \mu = 6$$

$$H_1 : \mu < 6$$

$$\alpha = 0.05$$

Using Minitab Wilcoxon signed-rank t test

	N	Test	Statistic	P	Estimated
Y	15	13	19.0	0.035	5.000

Reject H_0

The Wilcoxon signed-rank test applies to symmetric continuous distributions. The test to applies to the mean of the distribution.

- 15-51. 1. The parameters of interest are the mean volumes

$$2. H_0 : \mu_1 = \mu_2$$

$$3. H_1 : \mu_1 \neq \mu_2$$

$$4. \alpha = 0.01$$

5. The test statistic is $w_2 = \frac{(n_1 + n_2)(n_1 + n_2 + 1)}{2} - w_1$

6. We will reject H_0 if $w \leq W_{0.01}^* = 71$, since $\alpha=0.01$ and $n_1=10$ and $n_2=10$, the value in Appendix A, Table IX.

7. $w_1 = 42$ and $w_2 = 78$ and so since 42 is less than 78, we reject H_0

8. Conclusion: reject H_0

15-57. Kruskal-Wallis Test on VOLUME

TEMPERAT	N	Median	Ave Rank	Z
70	5	1245	12.4	2.69
75	5	1220	7.9	-0.06
80	5	1170	3.7	-2.63
Overall	15		8.0	

H = 9.46 DF = 2 P = 0.009
H = 9.57 DF = 2 P = 0.008 (adjusted for ties)
Reject H_0 , P-value=0.0009