Chapter 16 Selected Problem Solutions

<u>Section 16-5</u>

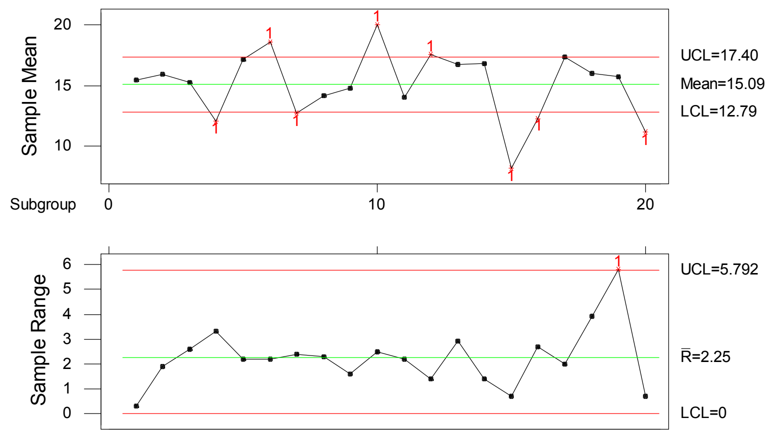
16-3 a)

```
                    X-bar and Range - Initial Study
                         Charting Problem 16-3
X-bar                                    | Range
-----                                    | -----
UCL:  +    3.0 sigma = 17.4              | UCL:  +    3.0 sigma =  5.792
Centerline           = 15.09             | Centerline           =  2.25
LCL:  -    3.0 sigma = 12.79             | LCL:  -    3.0 sigma =  0
                                         |

Test Results: X-bar One point more than 3.00 sigmas from center line.
Test Failed at points: 4 6 7 10 12 15 16 20

Test Results for R Chart:One point more than 3.00 sigmas from center line.
Test Failed at points: 19
```

Xbar/R Chart for x1-x3



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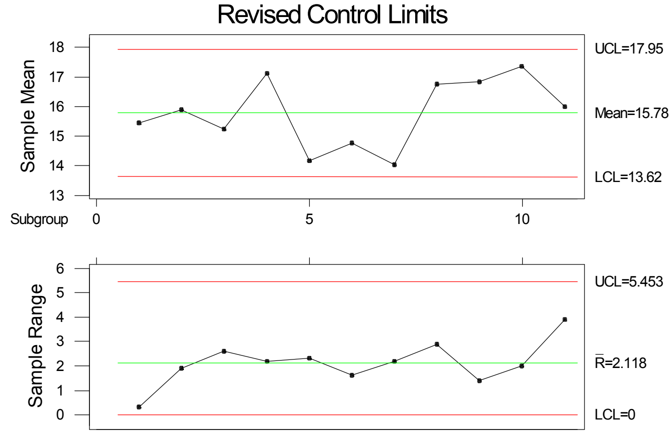
b. Removed points 4, 6, 7, 10, 12, 15, 16, 19, and 20 and revised the control limits
The control limits are not as wide after being revised X-bar UCL=17.96, CL=15.78 LCL=13.62 and R UCL = 5.453, R-bar=2.118, LCL=0.
The X-bar control moved down.

Xbar/R Chart for x1-x3
Revised Control Limits



c)

```
                         X-bar and StDev - Initial Study
                           Charting Problem 16-3
X-bar                                     | StDev
-----                                     | -----
UCL:  +    3.0 sigma = 17.42              | UCL:  +    3.0 sigma =  3.051
Centerline           = 15.09              | Centerline           =  1.188
LCL:  -    3.0 sigma = 12.77              | LCL:  -    3.0 sigma =  0
                                          |

Test Results: X-bar One point more than 3.00 sigmas from center line.
Test Failed at points: 4 6 7 10 12 15 16 20

Test Results for S Chart:One point more than 3.00 sigmas from center line.
Test Failed at points: 19
```

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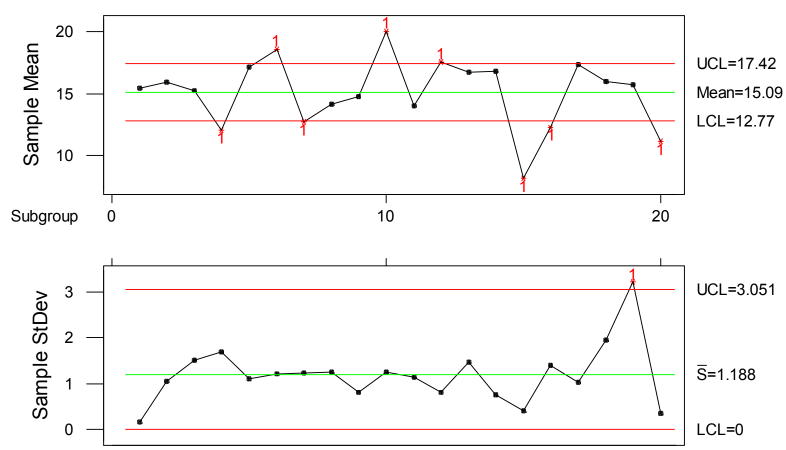
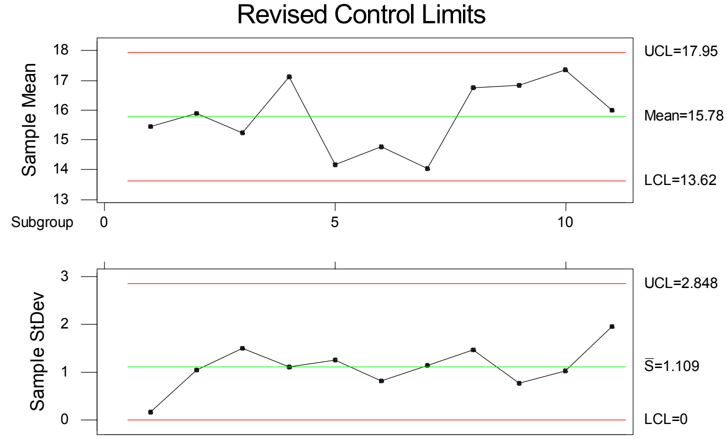
Xbar/S Chart for x1-x3



Removed points 4, 6, 7, 10, 12, 15, 16, 19, and 20 and revised the control limits
The control limits are not as wide after being revised X-bar UCL=17.95, CL=15.78
LCL=13.62 and S UCL = 2.848, S-bar=1.109, LCL=0.
The X-bar control moved down.

Xbar/S Chart for x1-x3
Revised Control Limits



16-5. a) $\bar{\bar{x}} = \dfrac{7805}{35} = 223 \qquad \bar{r} = \dfrac{1200}{35} = 34.286$

$\bar{x}$ *chart*

$$UCL = CL + A_2\bar{r} = 223 + 0.577(34.286) = 242.78$$
$$CL = 223$$
$$LCL = CL - A_2\bar{r} = 223 - 0.577(34.286) = 203.22$$

R *chart*

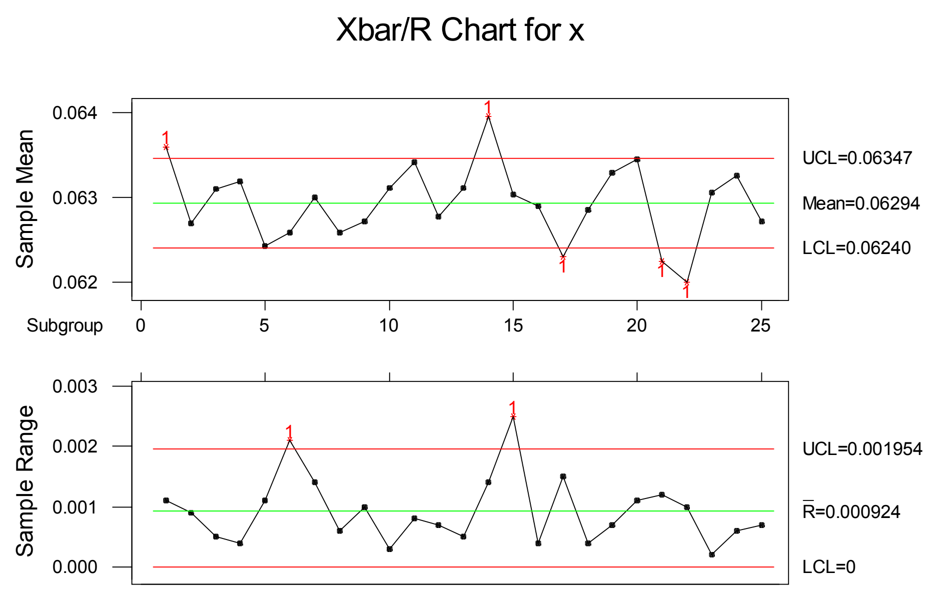
$$UCL = D_4\bar{r} = 2.115(34.286) = 72.51$$
$$CL = 34.286$$
$$LCL = D_3\bar{r} = 0(34.286) = 0$$

b)

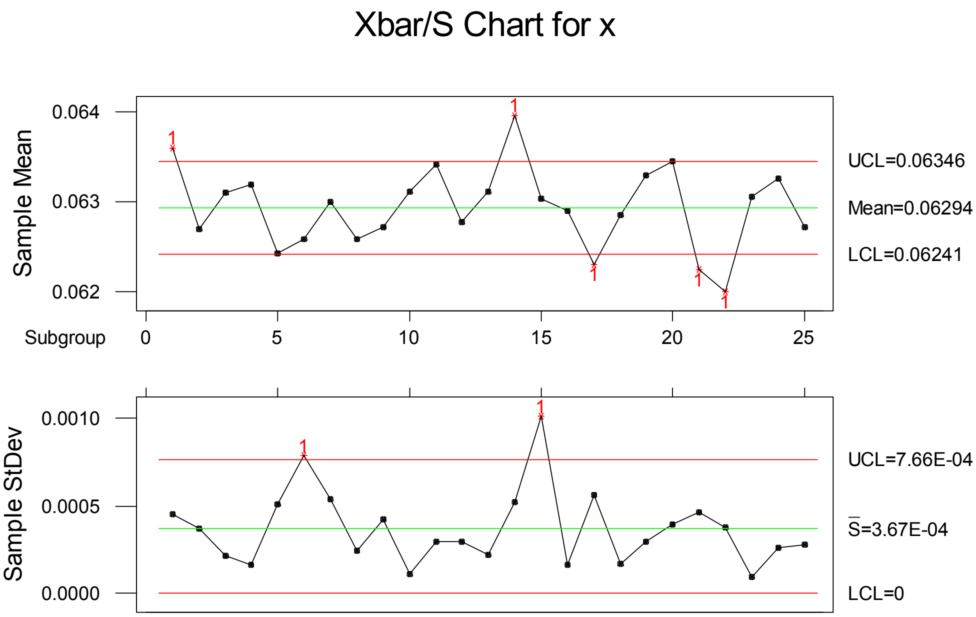
$$\hat{\mu} = \bar{\bar{x}} = 223$$
$$\hat{\sigma} = \frac{\bar{r}}{d_2} = \frac{34.286}{2.326} = 14.74$$

16-7 a)

```
                      X-bar and Range - Initial Study
                          Charting Problem 16-7
X-bar                                     | Range
-----                                     | -----
UCL:   +     3.0 sigma = 0.0634706        | UCL:   +     3.0 sigma = 1.95367E-3
Centerline             = 0.0629376        | Centerline             = 9.24E-4
LCL:   -     3.0 sigma = 0.0624046        | LCL:   -     3.0 sigma = 0
                                          |
out of limits = 5                         | out of limits = 2
---------------------------------------------------------------------------------
Chart: Both      Normalize: No
25 subgroups, size 5                         0 subgroups excluded
Estimated
```



b)



c) There are several points out of control. The control limits need to be revised.
The points are 1, 5, 14,17, 20,21, and 22; or outside the control limits of the R chart: 6 and 15

<u>Section 16-6</u>

16-9. a)

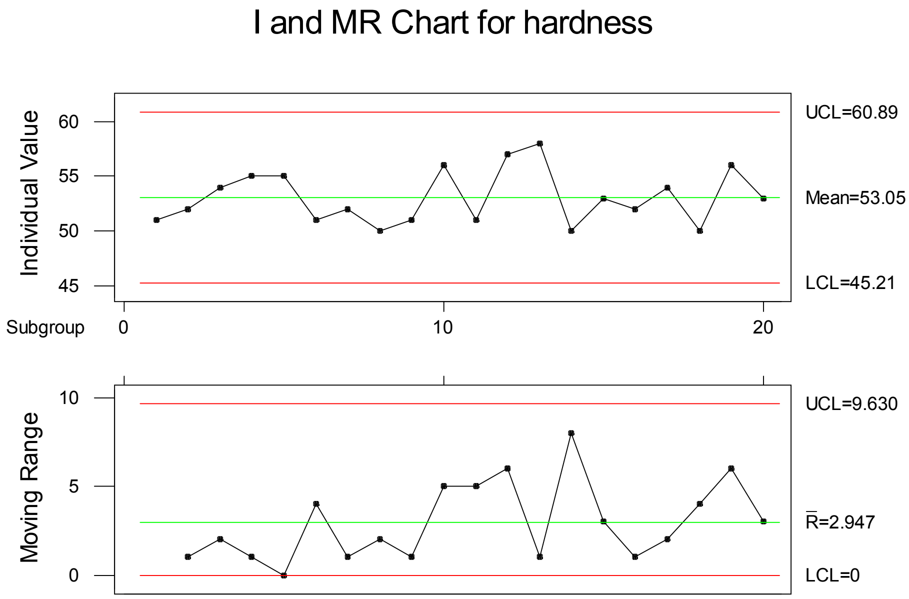
```
                    Individuals and MR(2) - Initial Study
------------------------------------------------------------------------------
                            Charting Problem 15-8

Ind.x                                  | MR(2)
-----                                  | -----
UCL:  +    3.0 sigma = 60.8887         | UCL:  +    3.0 sigma = 9.63382
Centerline           = 53.05           | Centerline           = 2.94737
LCL:  -    3.0 sigma = 45.2113         | LCL:  -    3.0 sigma = 0
                                       |
out of limits = 0                      | out of limits = 0
------------------------------------------------------------------------------
Chart: Both      Normalize: No

   20 subgroups, size 1                    0 subgroups excluded

Estimated
process mean  = 53.05
process sigma =  2.61292
mean MR       =  2.94737
```

There are no points beyond the control limits. The process appears to be in control.



b)

$$\hat{\mu} = \bar{x} = 53.05$$

$$\hat{\sigma} = \frac{\overline{mr}}{d_2} = \frac{2.94737}{1.128} = 2.613$$

Section 16-7

16-15. a) Assuming a normal distribution with $\hat{\mu}$ = 0.14.510 and $\hat{\sigma} = \frac{\bar{r}}{d_2} = \frac{0.344}{2.326} = 0.148$

$$\begin{aligned} P(X < LSL) &= P\left(Z < \frac{LSL - \hat{\mu}}{\hat{\sigma}}\right) \\ &= P\left(Z < \frac{14.00 - 14.51}{0.148}\right) \\ &= P(Z < -3.45) \\ &= 1 - P(Z < 3.45) \\ &= 1 - 0.99972 \\ &= 0.00028 \end{aligned}$$

$$
\begin{aligned}
P(X > USL) &= P\left(Z > \frac{USL-\hat{\mu}}{\hat{\sigma}}\right) \\
&= P\left(Z > \frac{15.00-14.51}{0.148}\right) \\
&= P(Z > 3.31) \\
&= 1 - P(Z < 3.31) \\
&= 1 - 0.99953 \\
&= 0.00047
\end{aligned}
$$

Therefore, the proportion nonconforming is given by
P(X<LSL) + P(X>USL) =0.00028 + 0.00047 = 0.00075

b)

$$
PCR = \frac{USL-LSL}{6(\hat{\sigma})} = \frac{15.00-14.00}{6(0.148)} = 1.13
$$

$$
\begin{aligned}
PCR_K &= \min\left[\frac{USL-\bar{\bar{x}}}{3\hat{\sigma}}, \frac{\bar{\bar{x}}-LSL}{3\hat{\sigma}}\right] \\
&= \min\left[\frac{15.00-14.51}{3(0.148)}, \frac{14.51-14.00}{3(0.148)}\right] \\
&= \min[1.104, 1.15] \\
&= 1.104
\end{aligned}
$$

Since PCR exceeds unity, the natural tolerance limits lie inside the specification limits and very few defective units will be produced.

$PCR_K \cong PCR$ the process appears to be centered.

16-19 a) Assuming a normal distribution with $\hat{\mu}$ = 223 and $\hat{\sigma} = \frac{\bar{s}}{c_4} = \frac{13.58}{0.9213} = 14.74$

$$
\begin{aligned}
P(X < LSL) &= P\left(Z < \frac{LSL-\hat{\mu}}{\hat{\sigma}}\right) \\
&= P\left(Z < \frac{170-223}{14.74}\right) \\
&= P(Z < -3.60) \\
&= 0.00016
\end{aligned}
$$

$$
\begin{aligned}
P(X > USL) &= P\left(Z > \frac{USL - \hat{\mu}}{\hat{\sigma}}\right) \\
&= P\left(Z > \frac{270 - 223}{14.75}\right) \\
&= P(Z > 3.18) \\
&= 1 - P(Z < 3.18) \\
&= 1 - 0.99926 \\
&= 0.00074
\end{aligned}
$$

Probability of producing a part outside the specification limits is 0.00016+0.00074 = 0.0009

b

$$
PCR = \frac{USL - LSL}{6(\hat{\sigma})} = \frac{270 - 220}{6(14.75)} = 1.13
$$

$$
\begin{aligned}
PCR_K &= \min\left[\frac{USL - \bar{\bar{x}}}{3\hat{\sigma}}, \frac{\bar{\bar{x}} - LSL}{3\hat{\sigma}}\right] \\
&= \min\left[\frac{270 - 223}{3(14.75)}, \frac{223 - 170}{3(14.75)}\right] \\
&= \min[1.06, 1.19] \\
&= 1.06
\end{aligned}
$$

Since PCR exceeds unity, the natural tolerance limits lie inside the specification limits and very few defective units will be produced. The estimated proportion nonconforming is given by P(X<LSL) + P(X>USL) =0.00016 + 0.00074=0.0009

16-23. Assuming a normal distribution with $\hat{\mu}$ = 500.6 and $\hat{\sigma}$ = 17.17

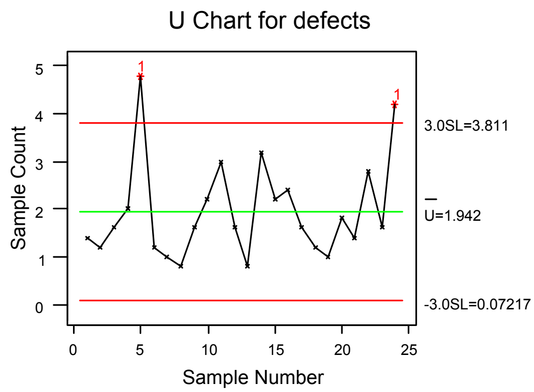
$$
PCR = \frac{USL - LSL}{6(\hat{\sigma})} = \frac{525 - 475}{6(17.17)} = 0.49
$$

$$
\begin{aligned}
PCR_K &= \min\left[\frac{USL - \bar{\bar{x}}}{3\hat{\sigma}}, \frac{\bar{\bar{x}} - LSL}{3\hat{\sigma}}\right] \\
&= \min\left[\frac{525 - 500.6}{3(17.17)}, \frac{500.6 - 475}{3(17.17)}\right] \\
&= \min[0.474, 0.50] \\
&= 0.474
\end{aligned}
$$

Since the process capability ratios are less than unity, the process capability appears to be poor.

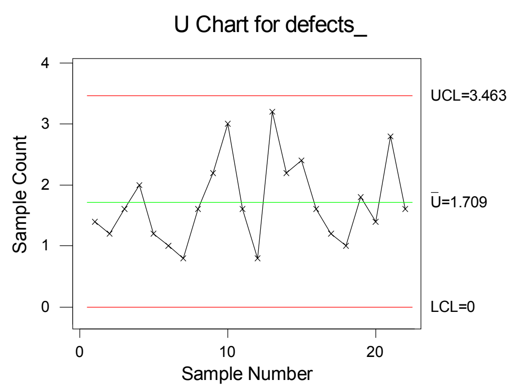
Section 16-8

16-25.



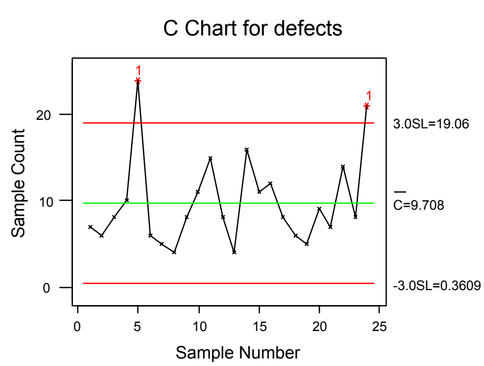
Samples 5 and 24 have out-of-control points. The limits need to be revised.

b)



The control limits are calculated without the out-of-control points. There are no points out of control for the revised limits.

16-27.



There are two points beyond the control limits. They are samples 5 and 24.
The U chart and the C chart both detected out-of-control points at samples 5 and 24.

Section 16-9

16-31. a) $\hat{\sigma}_{\bar{x}} = \dfrac{\hat{\sigma}}{\sqrt{n}} = \dfrac{2.4664}{\sqrt{5}} = 1.103$, $\mu = 36$

$$P(30.78 < \overline{X} < 37.404)$$
$$= P\left(\frac{30.78 - 36}{1.103} < \frac{\overline{X} - \mu}{\hat{\sigma}_{\bar{x}}} < \frac{37.404 - 36}{1.103}\right)$$
$$= P(-4.73 < Z < 1.27) = P(Z < 1.27) - P(Z < -4.73)$$
$$= 0.8980 - 0 = 0.8980$$

The probability that this shift will be detected on the next sample is p = 1–0.8980 = 0.1020.

b) $ARL = \dfrac{1}{p} = \dfrac{1}{0.102} = 9.8$

16-33. a) $\hat{\sigma} = \dfrac{\overline{R}}{d_2} = \dfrac{6.75}{2.059} = 3.28$ $\hat{\sigma}_{\bar{x}} = \dfrac{\hat{\sigma}}{\sqrt{n}} = \dfrac{3.28}{\sqrt{4}} = 1.64$, $\mu = 13$

$$P(5.795 < \overline{X} < 15.63)$$
$$= P\left(\frac{5.795 - 13}{1.64} < \frac{\overline{X} - \mu}{\sigma_{\bar{x}}} < \frac{15.63 - 13}{1.64}\right)$$
$$= P(-4.39 < Z < 1.60) = P(Z < 1.60) - P(Z < -4.39)$$
$$= 0.9452 - 0 = 0.9452$$

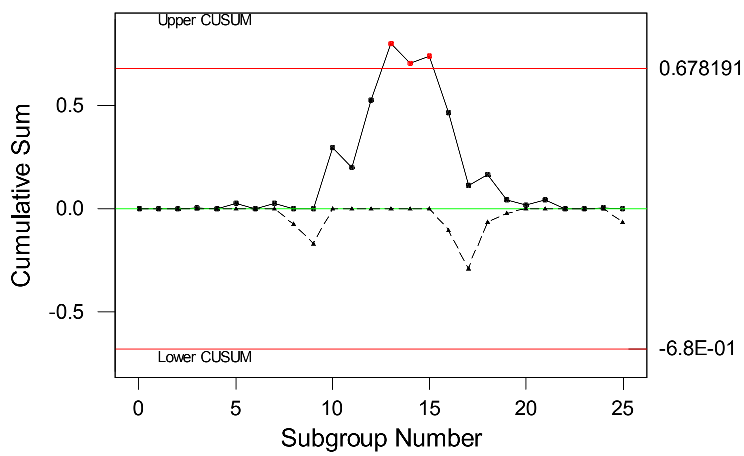
The probability that this shift will be detected on the next sample is p = 1–0.9452 = 0.0548.

b) $ARL = \dfrac{1}{p} = \dfrac{1}{0.0548} = 18.25$

Section 16-10

16-39. a) $\hat{\sigma}$ = 0.1695
b) The process appears to be out of control at the specified target level.

CUSUM Chart for diameter

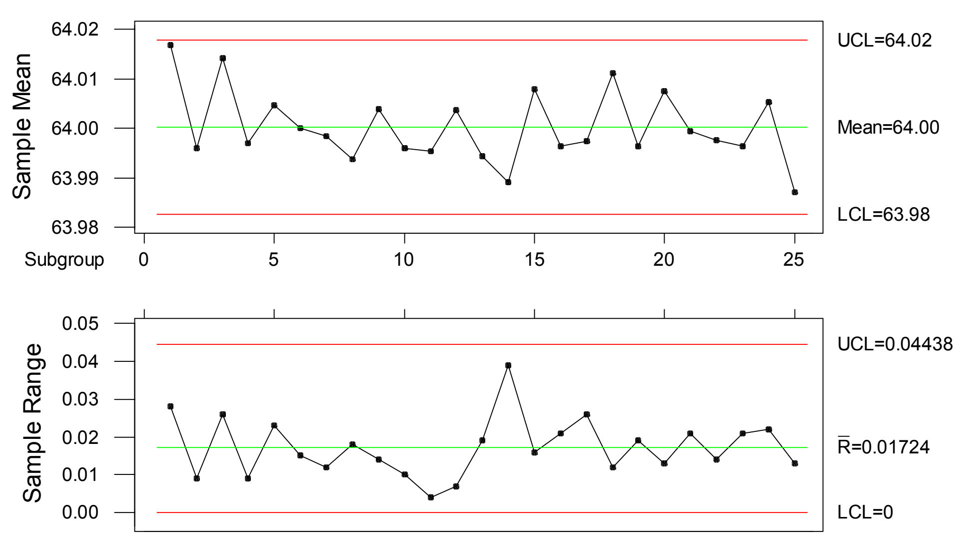


<u>Supplemental</u>

16-43. a)

```
                        X-bar and Range - Initial Study
--------------------------------------------------------------------------------
X-bar                                  | Range
----                                  | -----
UCL:  +    3.0 sigma = 64.0181         | UCL:  +    3.0 sigma = 0.0453972
Centerline           = 64              | Centerline           = 0.01764
LCL:  -    3.0 sigma = 63.982          | LCL:  -    3.0 sigma = 0
                                       |
out of limits = 0                      | out of limits = 0
--------------------------------------------------------------------------------
Chart: Both      Normalize: No
Estimated
process mean  = 64
process sigma =  0.0104194
mean Range    =  0.01764
```

Xbar/R Chart for diameter



The process is in control.

b) $\hat{\mu} = \bar{\bar{x}} = 64$ $\quad \hat{\sigma} = \dfrac{\overline{R}}{d_2} = \dfrac{0.01764}{1.693} = 0.0104$

c) $PCR = \dfrac{USL - LSL}{6\hat{\sigma}} = \dfrac{64.02 - 63.98}{6(0.0104)} = 0.641$

The process does not meet the minimum capability level of PCR ≥ 1.33.

d)

$$PCR_k = \min\left[\frac{USL - \bar{\bar{x}}}{3\hat{\sigma}}, \frac{\bar{\bar{x}} - LSL}{3\hat{\sigma}}\right] = \min\left[\frac{64.02 - 64}{3(0.0104)}, \frac{64 - 63.98}{3(0.0104)}\right]$$
$$= \min[0.641, 0.641] = 0.641$$

e) In order to make this process a "six-sigma process", the variance σ^2 would have to be decreased such that $PCR_k = 2.0$. The value of the variance is found by solving $PCR_k = \dfrac{\bar{\bar{x}} - LSL}{3\sigma} = 2.0$ for σ:

$$\frac{64 - 61}{3\sigma} = 2.0$$
$$6\sigma = 64. - 61$$
$$\sigma = \frac{64. - 61}{6} = 0.50$$

Therefore, the process variance would have to be decreased to $\sigma^2 = (0.50)^2 = 0.025$.

f) $\hat{\sigma}_{\bar{x}} = 0.0104$

$$P(63.98 < X < 64.02)$$
$$= P\left(\frac{63.98 - 64.01}{0.0104} < \frac{X - \mu}{\sigma_x} < \frac{64.02 - 64.01}{0.0104}\right)$$
$$= P(-2.88 < Z < 0.96) = P(Z < 0.96) - P(Z < -2.88)$$
$$= 0.8315 - 0.0020 = 0.8295$$

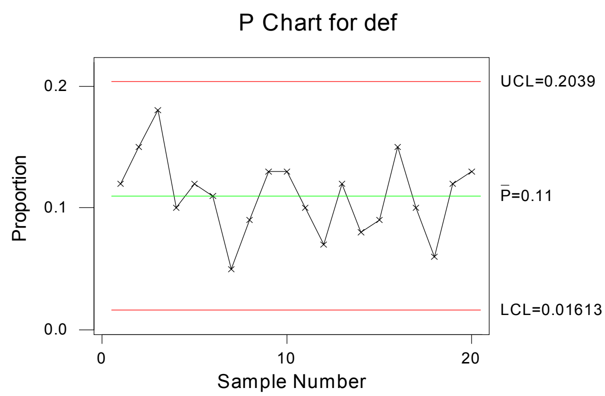
The probability that this shift will be detected on the next sample is p = 1–0.8295 = 0.1705

$$ARL = \frac{1}{p} = \frac{1}{0.1705} = 5.87$$

16-45. a)

P Chart - Initial Study

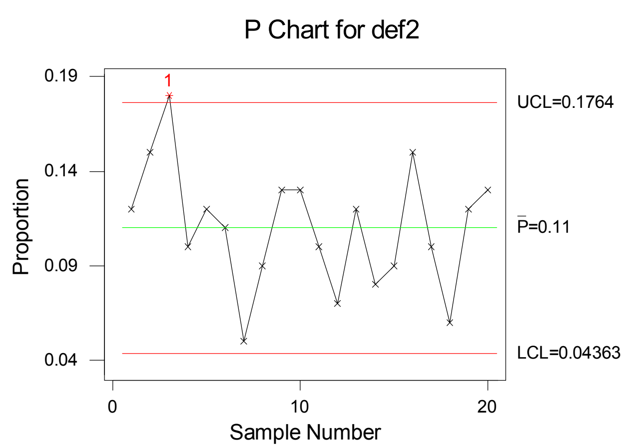
```
P Chart
-----
UCL:  +    3.0 sigma = 0.203867
Centerline           = 0.11
LCL:  -    3.0 sigma = 0.0161331
out of limits = 0
Estimated
mean P   = 0.11
sigma    = 0.031289
```



There are no points beyond the control limits. The process is in control.

b)

```
                              P Chart - Initial Study
                              Sample Size, n = 200
P Chart
-----
UCL:  +    3.0 sigma = 0.176374
Centerline           = 0.11
LCL:  -    3.0 sigma = 0.0436261
out of limits = 1
Estimated
mean P  = 0.11
sigma   = 0.0221246
```

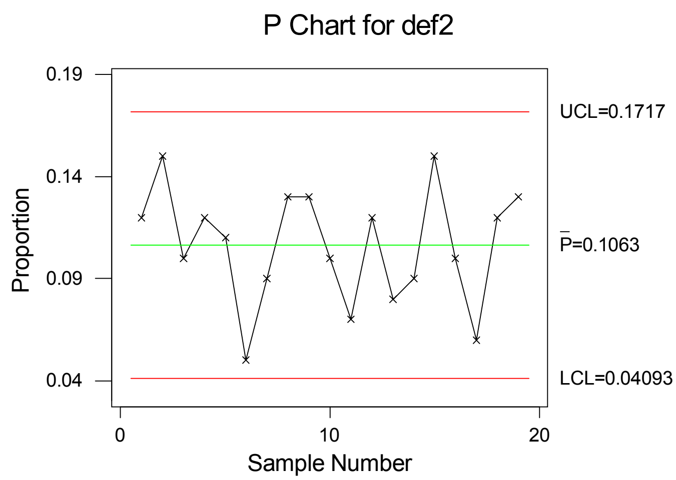


There is one point beyond the upper control limit. The process is out of control. The revised limits are:

```
                              P Chart - Revised Limits
                                Sample Size, n = 200
P Chart
-----
UCL:  +    3.0 sigma = 0.171704
Centerline           = 0.106316
LCL:  -    3.0 sigma = 0.0409279
out of limits = 0
Estimated
mean P  = 0.106316
```

```
sigma    = 0.021796
```

There are no points beyond the control limits. The process is now in control.



c) A larger sample size with the same number of defective items will result in more narrow control limits. The control limits corresponding to the larger sample size are more sensitive.

16-49. ARL = 1/p where p is the probability a point falls outside the control limits.

a) $\mu = \mu_0 + \sigma$ and n = 1

$$p = P(\overline{X} > UCL) + P(\overline{X} < LCL)$$

$$= P\left(Z > \frac{\mu_0 + \frac{3\sigma}{\sqrt{n}} - \mu_0 - \sigma}{\sigma / \sqrt{n}} \right) + P\left(Z < \frac{\mu_0 - \frac{3\sigma}{\sqrt{n}} - \mu_0 - \sigma}{\sigma / \sqrt{n}} \right)$$

$$= P(Z > 3 - \sqrt{n}) + P(Z < -3 - \sqrt{n})$$

$$= P(Z > 2) + P(Z < -4) \qquad when \quad n = 1$$

$$= 1 - P(Z < 2) + [1 - P(Z < 4)] \quad = 1 - 0.97725 + [1 - 1] \quad = 0.02275$$

Therefore, ARL = 1/p = 1/0.02275 = 43.9.

b) $\mu = \mu_0 + 2\sigma$

$$P(\overline{X} > UCL) + P(\overline{X} < LCL)$$
$$= P\left(Z > \frac{\mu_0 + \frac{3\sigma}{\sqrt{n}} - \mu_0 - 2\sigma}{\sigma / \sqrt{n}} \right) + P\left(Z < \frac{\mu_0 - \frac{3\sigma}{\sqrt{n}} - \mu_0 - 2\sigma}{\sigma / \sqrt{n}} \right)$$
$$= P(Z > 3 - 2\sqrt{n}) + P(Z < -3 - 2\sqrt{n})$$
$$= P(Z > 1) + P(Z < -5) \qquad when \quad n = 1$$
$$= 1 - P(Z < 1) + [1 - P(Z < 5)]$$
$$= 1 - 0.84134 + [1 - 1]$$
$$= 0.15866$$

Therefore, ARL = 1/p = 1/0.15866 = 6.30.

c) $\mu = \mu_0 + 3\sigma$

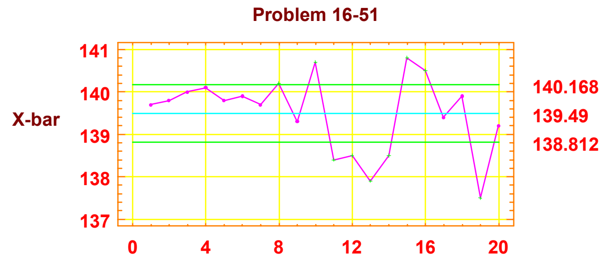
$$P(\overline{X} > UCL) + P(\overline{X} < LCL)$$
$$= P\left(Z > \frac{\mu_0 + \frac{3\sigma}{\sqrt{n}} - \mu_0 - 3\sigma}{\sigma / \sqrt{n}} \right) + P\left(Z < \frac{\mu_0 - \frac{3\sigma}{\sqrt{n}} - \mu_0 - 3\sigma}{\sigma / \sqrt{n}} \right)$$
$$= P(Z > 3 - 3\sqrt{n}) + P(Z < -3 - 3\sqrt{n})$$
$$= P(Z > 0) + P(Z < -6) \qquad when \quad n = 1$$
$$= 1 - P(Z < 0) + [1 - P(Z < 6)] = 1 - 0.50 + [1 - 1] = 0.50$$

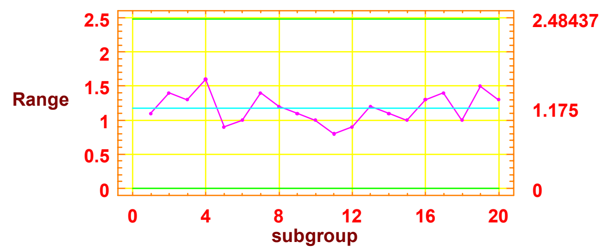
Therefore, ARL = 1/p = 1/0.50 = 2.00.

d) The ARL is decreasing as the magnitude of the shift increases from σ to 2σ to 3σ. The ARL will decrease as the magnitude of the shift increases since a larger shift is more likely to be detected earlier than a smaller shift.

16-51. a)

```
                     X-bar and Range - Initial Study
                            Charting xbar
X-bar                                   | Range
-----                                   | -----
UCL:  +    3.0 sigma = 140.168          | UCL:  +    3.0 sigma = 2.48437
Centerline           = 139.49           | Centerline           = 1.175
LCL:  -    3.0 sigma = 138.812          | LCL:  -    3.0 sigma = 0
out of limits = 9                       | out of limits = 0
Estimated
process mean  = 139.49
process sigma =   0.505159
mean Range    =   1.175
```





There are points beyond the control limits. The process is out of control. The points are 4, 8, 10, 13, 15, 16, and 19.

b) Revised control limits are given in the table below:

```
                         X-bar and Range - Initial Study
                                 Charting Xbar
X-bar                                      | Range
-----                                      | -----
UCL:  +    3.0 sigma = 140.518             | UCL:  +    3.0 sigma = 2.60229
Centerline           = 139.808             | Centerline           = 1.23077
LCL:  -    3.0 sigma = 139.098             | LCL:  -    3.0 sigma = 0
out of limits = 0                          | out of limits = 0
Estimated
process mean  = 139.808
process sigma =   0.529136
mean Range    =   1.23077
```

z

There are no points beyond the control limits the process is now in control.

The process standard deviation estimate is given by $\hat{\sigma} = \dfrac{\overline{R}}{d_2} = \dfrac{1.23077}{2.326} = 0.529$

c) $PCR = \dfrac{USL - LSL}{6\hat{\sigma}} = \dfrac{142 - 138}{6(0.529)} = 1.26$

$$PCR_k = \min\left[\frac{USL - \overline{\overline{x}}}{3\hat{\sigma}}, \frac{\overline{\overline{x}} - LSL}{3\hat{\sigma}}\right]$$

$$= \min\left[\frac{142 - 139.808}{3(0.529)}, \frac{139.808 - 138}{3(0.529)}\right]$$

$$= \min[1.38, 1.14]$$

$$= 1.14$$

Since PCR exceeds unity, the natural tolerance limits lie inside the specification limits and very few defective units will be produced.

PCR is slightly larger than PCR_k indicating that the process is somewhat off center.

d) In order to make this process a “six-sigma process”, the variance σ^2 would have to be decreased such that $PCR_k = 2.0$. The value of the variance is found by solving $PCR_k = \dfrac{\overline{\overline{x}} - LSL}{3\sigma} = 2.0$ for σ:

$$\frac{139.808-138}{3\sigma}=2.0$$

$$6\sigma=139.808-138$$

$$\sigma=\frac{139.808-138}{6}$$

$$\sigma=0.3013$$

Therefore, the process variance would have to be decreased to $\sigma^2 = (0.3013)^2 = 0.091$.

e) $\hat{\sigma}_{\overline{x}} = 0.529$

$$p = P(139.098 < X < 140.518 \mid \mu = 139.7)$$

$$= P\left(\frac{139.098-139.7}{0.529} < \frac{X-\mu}{\sigma_x} < \frac{140.518-139.7}{0.529}\right)$$

$$= P(-1.14 < Z < 1.55) = P(Z < 1.55) - P(Z < -1.14)$$

$$= P(Z < 1.55) - [1 - P(Z < 1.14)] = 0.93943 - [1 - 0.87285] = 0.8123$$

The probability that this shift will be detected on the next sample is 1–p = 1–0.8123 = 0.1877.

$$\text{ARL} = \frac{1}{1-\text{p}} = \frac{1}{0.1877} = 5.33$$