

Applied Statistics and Probability for Engineers

Third Edition

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To:

Meredith, Neil, Colin, and Cheryl

Rebecca, Elisa, George, and Taylor

Preface

This is an introductory textbook for a first course in applied statistics and probability for undergraduate students in engineering and the physical or chemical sciences. These individuals play a significant role in designing and developing new products and manufacturing systems and processes, and they also improve existing systems. Statistical methods are an important tool in these activities because they provide the engineer with both descriptive and analytical methods for dealing with the variability in observed data. Although many of the methods we present are fundamental to statistical analysis in other disciplines, such as business and management, the life sciences, and the social sciences, we have elected to focus on an engineering-oriented audience. We believe that this approach will best serve students in engineering and the chemical/physical sciences and will allow them to concentrate on the many applications of statistics in these disciplines. We have worked hard to ensure that our examples and exercises are engineering- and science-based, and in almost all cases we have used examples of real data—either taken from a published source or based on our consulting experiences.

We believe that engineers in all disciplines should take at least one course in statistics. Unfortunately, because of other requirements, most engineers will only take one statistics course. This book can be used for a single course, although we have provided enough material for two courses in the hope that more students will see the important applications of statistics in their everyday work and elect a second course. We believe that this book will also serve as a useful reference.

ORGANIZATION OF THE BOOK

We have retained the relatively modest mathematical level of the first two editions. We have found that engineering students who have completed one or two semesters of calculus should have no difficulty reading almost all of the text. It is our intent to give the reader an understanding of the methodology and how to apply it, not the mathematical theory. We have made many enhancements in this edition, including reorganizing and rewriting major portions of the book.

Perhaps the most common criticism of engineering statistics texts is that they are too long. Both instructors and students complain that it is impossible to cover all of the topics in the book in one or even two terms. For authors, this is a serious issue because there is great variety in both the content and level of these courses, and the decisions about what material to delete without limiting the value of the text are not easy. After struggling with these issues, we decided to divide the text into two components; a set of core topics, many of which are most

likely to be covered in an engineering statistics course, and a set of supplementary topics, or topics that will be useful for some but not all courses. The core topics are in the printed book, and the complete text (both core and supplementary topics) is available on the CD that is included with the printed book. Decisions about topics to include in print and which to include only on the CD were made based on the results of a recent survey of instructors.

The *Interactive e-Text* consists of the complete text and a wealth of additional material and features. The text and links on the CD are navigated using Adobe Acrobat™. The links within the *Interactive e-Text* include the following: (1) from the Table of Contents to the selected *eText* sections, (2) from the Index to the selected topic within the *e-Text*, (3) from reference to a figure, table, or equation in one section to the actual figure, table, or equation in another section (all figures can be enlarged and printed), (4) from end-of-chapter Important Terms and Concepts to their definitions within the chapter, (5) from in-text **boldfaced terms** to their corresponding Glossary definitions and explanations, (6) from in-text references to the corresponding Appendix tables and charts, (7) from boxed-number end-of-chapter exercises (essentially most odd-numbered exercises) to their answers, (8) from some answers to the complete problem solution, and (9) from the opening splash screen to the textbook Web site.

Chapter 1 is an introduction to the field of statistics and how engineers use statistical methodology as part of the engineering problem-solving process. This chapter also introduces the reader to some engineering applications of statistics, including building empirical models, designing engineering experiments, and monitoring manufacturing processes. These topics are discussed in more depth in subsequent chapters.

Chapters 2, 3, 4, and 5 cover the basic concepts of probability, discrete and continuous random variables, probability distributions, expected values, joint probability distributions, and independence. We have given a reasonably complete treatment of these topics but have avoided many of the mathematical or more theoretical details.

Chapter 6 begins the treatment of statistical methods with random sampling; data summary and description techniques, including stem-and-leaf plots, histograms, box plots, and probability plotting; and several types of time series plots. Chapter 7 discusses point estimation of parameters. This chapter also introduces some of the important properties of estimators, the method of maximum likelihood, the method of moments, sampling distributions, and the central limit theorem.

Chapter 8 discusses interval estimation for a single sample. Topics included are confidence intervals for means, variances or standard deviations, and proportions and prediction and tolerance intervals. Chapter 9 discusses hypothesis tests for a single sample. Chapter 10 presents tests and confidence intervals for two samples. This material has been extensively rewritten and reorganized. There is detailed information and examples of methods for determining appropriate sample sizes. We want the student to become familiar with how these techniques are used to solve real-world engineering problems and to get some understanding of the concepts behind them. We give a logical, heuristic development of the procedures, rather than a formal mathematical one.

Chapters 11 and 12 present simple and multiple linear regression. We use matrix algebra throughout the multiple regression material (Chapter 12) because it is the only easy way to understand the concepts presented. Scalar arithmetic presentations of multiple regression are awkward at best, and we have found that undergraduate engineers are exposed to enough matrix algebra to understand the presentation of this material.

Chapters 13 and 14 deal with single- and multifactor experiments, respectively. The notions of randomization, blocking, factorial designs, interactions, graphical data analysis, and fractional factorials are emphasized. Chapter 15 gives a brief introduction to the methods and applications of nonparametric statistics, and Chapter 16 introduces statistical quality control, emphasizing the control chart and the fundamentals of statistical process control.

Each chapter has an extensive collection of exercises, including end-of-section exercises that emphasize the material in that section, supplemental exercises at the end of the chapter that cover the scope of chapter topics, and mind-expanding exercises that often require the student to extend the text material somewhat or to apply it in a novel situation. As noted above, answers are provided to most odd-numbered exercises and the *e-Text* contains complete solutions to selected exercises.

USING THE BOOK

This is a very flexible textbook because instructors' ideas about what should be in a first course on statistics for engineers vary widely, as do the abilities of different groups of students. Therefore, we hesitate to give too much advice but will explain how we use the book.

We believe that a first course in statistics for engineers should be primarily an applied statistics course, not a probability course. In our one-semester course we cover all of Chapter 1 (in one or two lectures); overview the material on probability, putting most of the emphasis on the normal distribution (six to eight lectures); discuss most of Chapters 6 through 10 on confidence intervals and tests (twelve to fourteen lectures); introduce regression models in Chapter 11 (four lectures); give an introduction to the design of experiments from Chapters 13 and 14 (six lectures); and present the basic concepts of statistical process control, including the Shewhart control chart from Chapter 16 (four lectures). This leaves about three to four periods for exams and review. Let us emphasize that the purpose of this course is to introduce engineers to how statistics can be used to solve real-world engineering problems, not to weed out the less mathematically gifted students. This course is not the "baby math-stat" course that is all too often given to engineers.

If a second semester is available, it is possible to cover the entire book, including much of the *e-Text* material, if appropriate for the audience. It would also be possible to assign and work many of the homework problems in class to reinforce the understanding of the concepts. Obviously, multiple regression and more design of experiments would be major topics in a second course.

USING THE COMPUTER

In practice, engineers use computers to apply statistical methods to solve problems. Therefore, we strongly recommend that the computer be integrated into the class. Throughout the book we have presented output from Minitab as typical examples of what can be done with modern statistical software. In teaching, we have used other software packages, including Statgraphics, JMP, and Statistica. We did not clutter up the book with examples from many different packages because how the instructor integrates the software into the class is ultimately more important than which package is used. All text data is available in electronic form on the *e-Text* CD. In some chapters, there are problems that we feel should be worked using computer software. We have marked these problems with a special icon in the margin.



In our own classrooms, we use the computer in almost every lecture and demonstrate how the technique is implemented in software as soon as it is discussed in the lecture. Student versions of many statistical software packages are available at low cost, and students can either purchase their own copy or use the products available on the PC local area networks. We have found that this greatly improves the pace of the course and student understanding of the material.

USING THE WEB

Additional resources for students and instructors can be found at www.wiley.com/college/montgomery/.

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Douglas C. Montgomery

George C. Runger

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